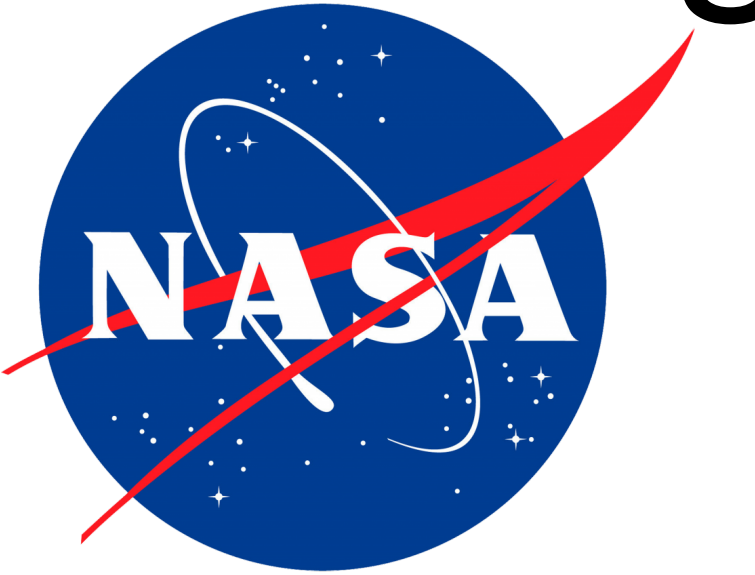


# Trajectory Planning for Autonomous Vehicles for Optimal Environmental Monitoring



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## Abstract

Our work considers real-time optimal trajectory planning for autonomous nonholonomic vehicles used in investigating environmental phenomena. We develop and implement an algorithm that generates optimal trajectories to either (1) reconstruct the field with minimal error, or (2) find the global maximum of the environmental field. Our algorithm uses Gaussian process priors to model the unknown field and Bayesian sequential experimental design methods, which involve developing utility functions that directly address the two operational goals.

## Methods

Notation

$\mathcal{S} \subset \mathbb{R}^2$  region of interest  
 $f: \mathcal{S} \rightarrow \mathbb{R}$  unknown function representing the environmental process  
 $\mathbf{q}: \mathbb{R}_{\geq 0} \rightarrow \mathcal{S}$  vehicle trajectory as a function of time  
 $\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_i = \mathbf{q}(i\Delta)$  sampling locations  
 $y_1, y_2, \dots$  observations acquired sequentially at  $\mathbf{s}_1, \mathbf{s}_2, \dots$

Model  $f$  using a Gaussian process prior

$$y_i = f(\mathbf{s}_i) + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma^2)$$

$$f \sim \mathcal{GP}(m(\mathbf{x}), C(\mathbf{x}, \mathbf{x}'))$$

The covariance function  $C(\mathbf{x}, \mathbf{x}')$  characterizes how related the process is at locations  $\mathbf{x}$  and  $\mathbf{x}'$ . The covariance functions we use are

Exponential  $C(\mathbf{x}, \mathbf{x}') = \tau^2 \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|}{\lambda}\right)$

Matérn  $C(\mathbf{x}, \mathbf{x}') = \frac{\tau^2 \|\mathbf{x} - \mathbf{x}'\|^\nu \mathbf{A}}{2^{\nu-1} \Gamma(\nu)} K_\nu(\|\mathbf{x} - \mathbf{x}'\| \mathbf{A})$

Then for any  $\mathbf{x} \in \mathcal{S}$ , the distribution of  $f(\mathbf{x})$  | data is Gaussian with

mean  $\mu_{\mathbf{y}_{1:n}}(\mathbf{x}) = m(\mathbf{x}) + \mathbf{c}_{1:n}^T(\mathbf{x}) \left[ \mathbf{C}_{1:n} + \sigma^2 \mathbf{I}_n \right]^{-1} [\mathbf{y}_{1:n} - \mathbf{m}_{1:n}]$

variance  $\kappa_{\mathbf{y}_{1:n}}^2(\mathbf{x}) = \tau^2 - \mathbf{c}_{1:n}^T(\mathbf{x}) \left[ \mathbf{C}_{1:n} + \sigma^2 \mathbf{I}_n \right]^{-1} \mathbf{c}_{1:n}(\mathbf{x})$

Bayesian experimental design maximizes an expected utility function to determine the locations for the next trajectory segment

$$\tilde{U}(\mathbf{s}) = \int U(\mathbf{s}, f) p(f | \text{data}) df$$

Expected utility functions for the two goals

(1)  $\tilde{U}(\mathbf{s}) = - \int_{\mathcal{S}} \left( \left[ \mu_{\mathbf{y}_{1:n}}(\mathbf{x}) - \mu_{\mathbf{y}_{1:n}}(\mathbf{s}) \right]^2 + \kappa_{\mathbf{y}_{1:n}}^2(\mathbf{s}) \right) d\mathbf{x}$

(2)  $\tilde{U}(\mathbf{s}) = [\mu_{\mathbf{y}_{1:n}}(\mathbf{s}) - y_{1:n}^{\max}] \Phi\left(\frac{\mu_{\mathbf{y}_{1:n}}(\mathbf{s}) - y_{1:n}^{\max}}{\kappa_{\mathbf{y}_{1:n}}(\mathbf{s})}\right) + \kappa_{\mathbf{y}_{1:n}}(\mathbf{s}) \phi\left(\frac{\mu_{\mathbf{y}_{1:n}}(\mathbf{s}) - y_{1:n}^{\max}}{\kappa_{\mathbf{y}_{1:n}}(\mathbf{s})}\right)$

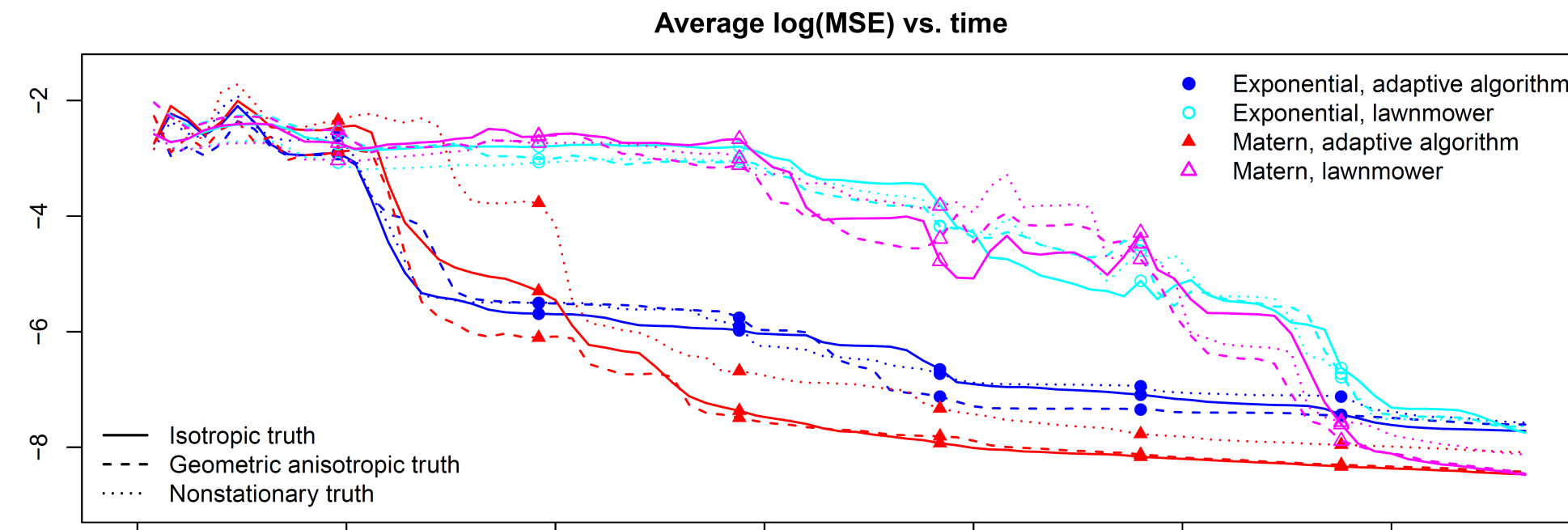
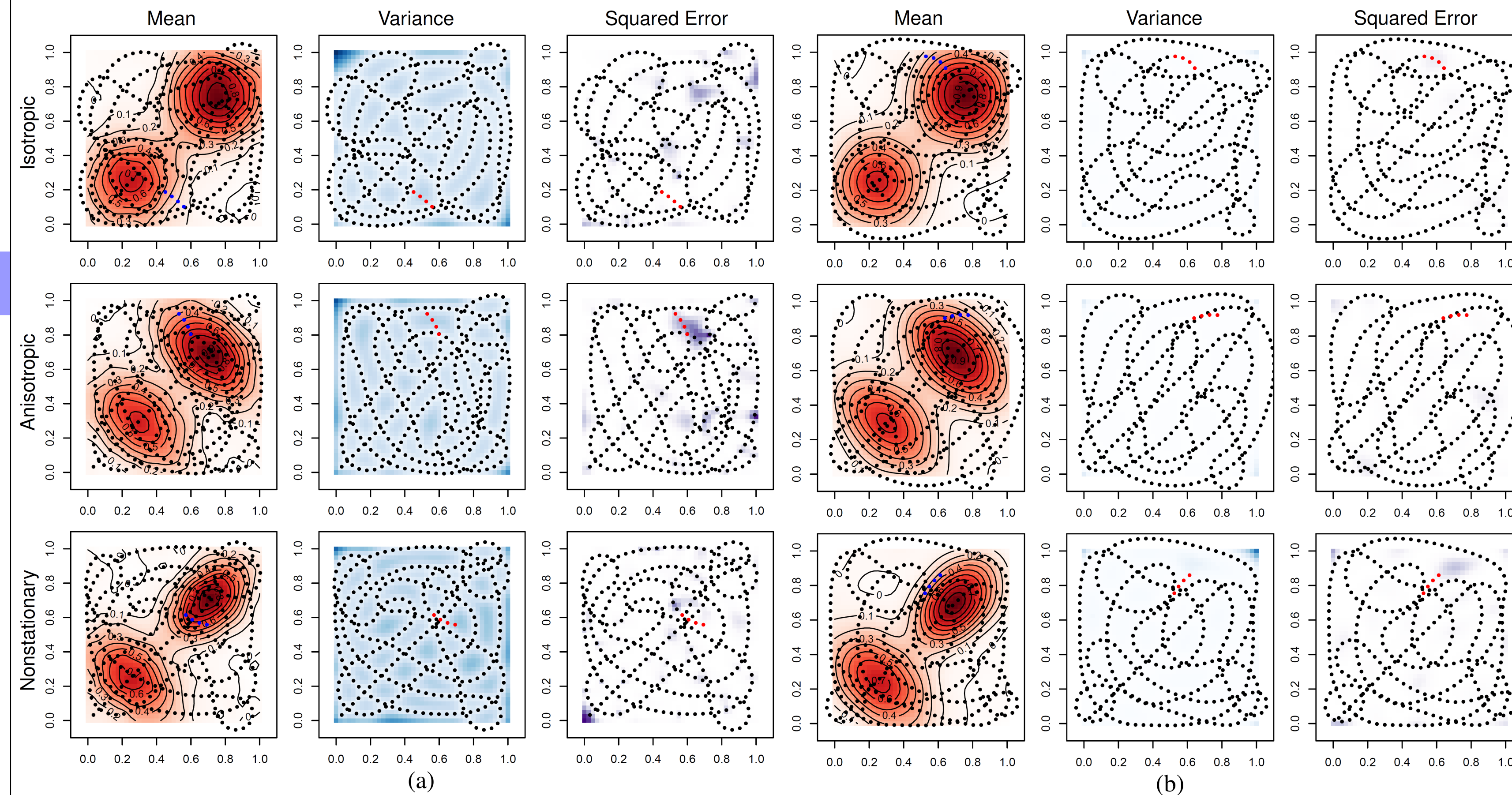
Parametrize the trajectory segments using circular arcs

$$\mathbf{q} = (\tilde{\mathbf{q}}_1, \tilde{\mathbf{q}}_2, \dots)$$

where  $\tilde{\mathbf{q}}_j(t) = \begin{bmatrix} \xi_{j,1} \cos(\xi_{j,2}t + \xi_{j,3}) + \xi_{j,4} \\ \xi_{j,1} \sin(\xi_{j,2}t + \xi_{j,3}) + \xi_{j,5} \end{bmatrix}$

## Simulation Results

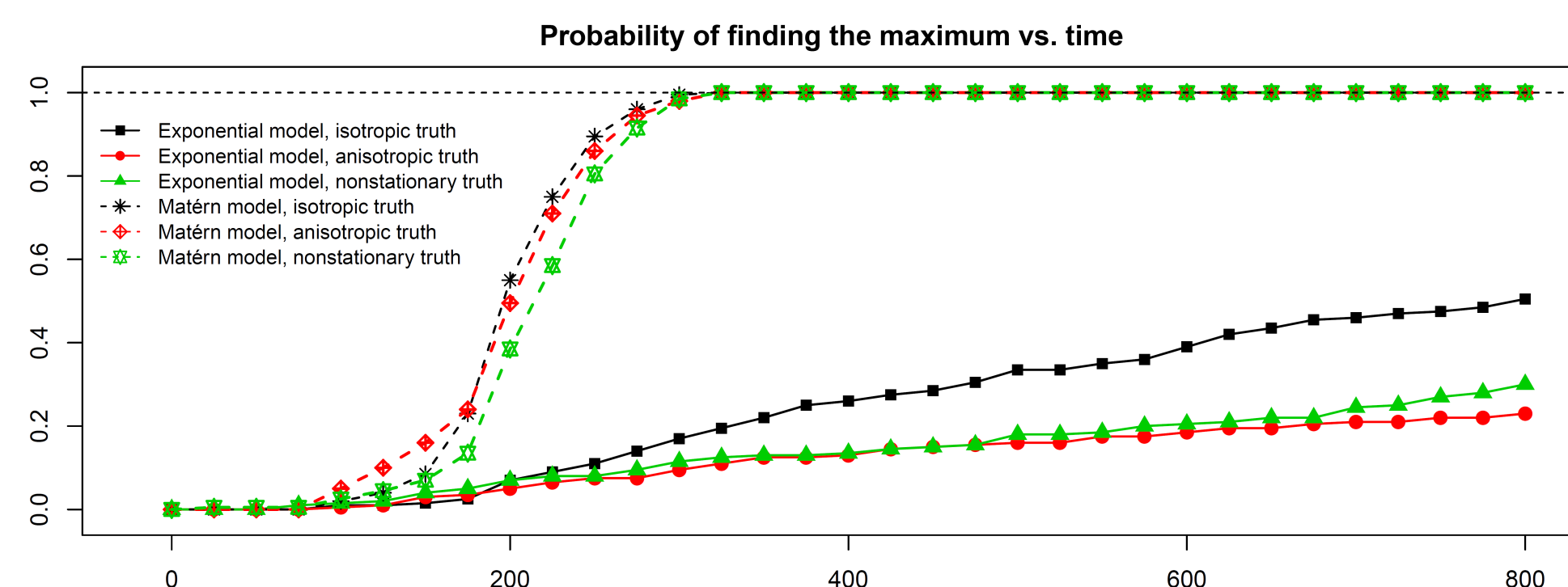
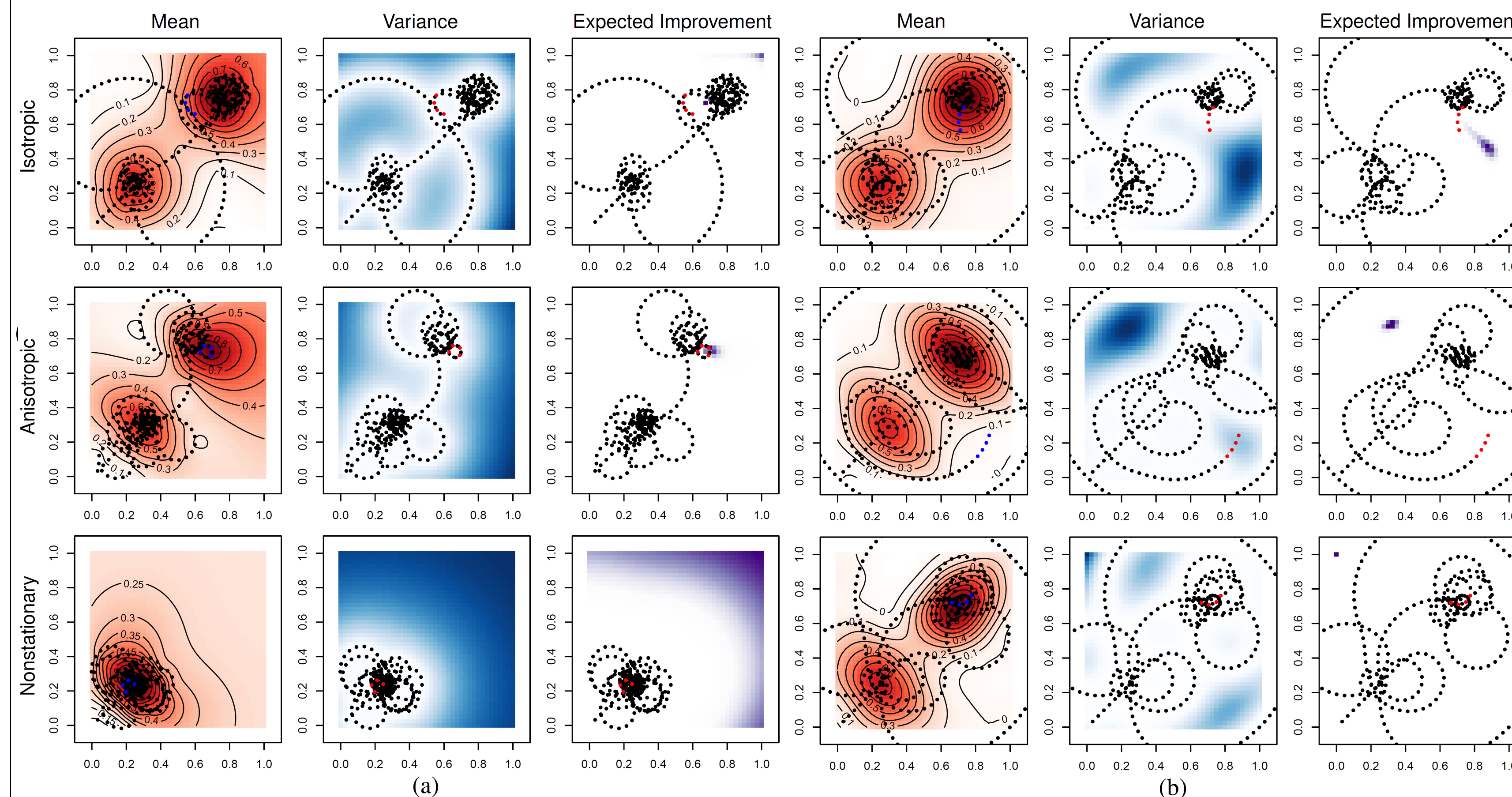
Minimum error reconstruction:



(ABOVE) Three different archetypes of spatial processes serve as the ground truth in our simulations: isotropic, geometric anisotropic, and nonstationary. After 332 observations, snapshots of the trajectory are plotted over the estimate of the process, variance of the estimate, and squared reconstruction error over the region when using the (a) exponential or (b) Matérn covariance function.

(LEFT) 200 simulations are run for each truth and each covariance type. The mean square reconstruction error over time is averaged over the 200 simulations and plotted. A corresponding set of simulations where the trajectory is a pre-planned "lawnmower" trajectory is plotted for comparison.

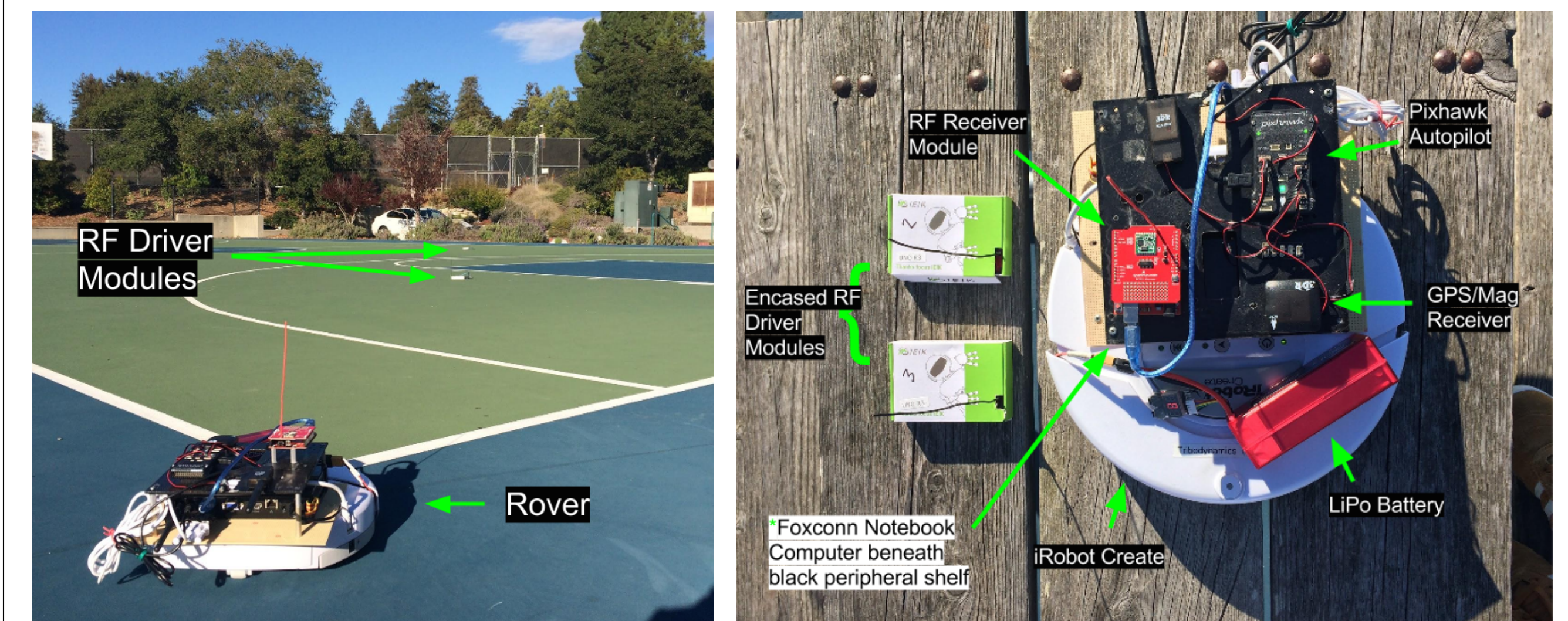
Finding the maximum:



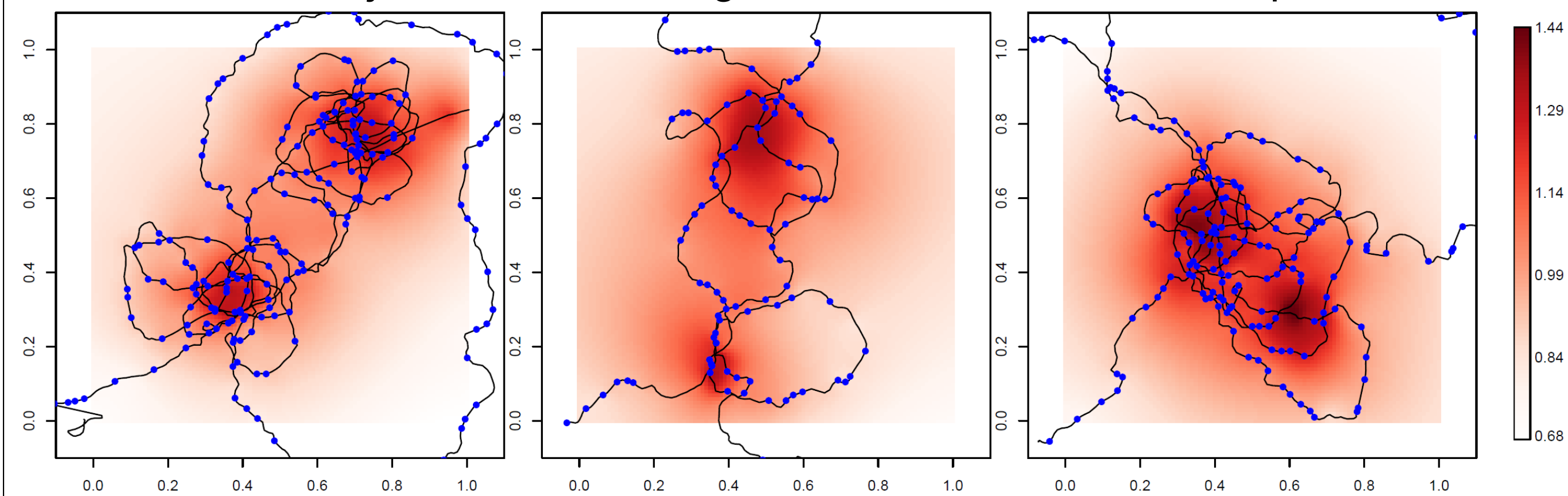
(ABOVE) For the three truth archetypes, snapshots of example trajectories taken at 332 observations computed using the (a) exponential or (b) Matérn covariance function. The trajectories are overlaid on the estimate of the process, variance of the estimate, and reward (expected improvement reflecting the goal of finding the maximum of the field) over the region.

(LEFT) The proportion of the simulations that have located the global maximum as a function of time.

## Field Tests

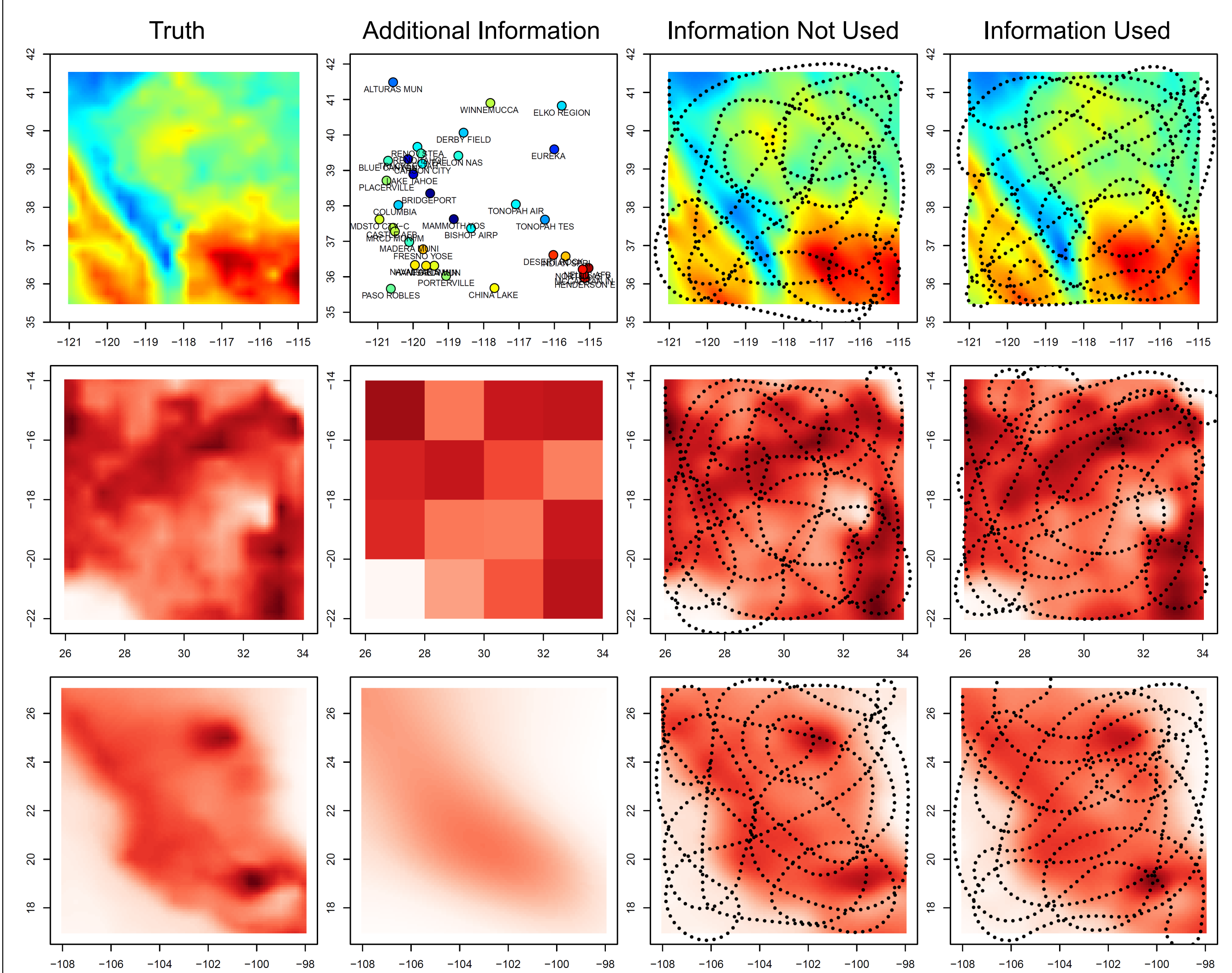


iRobot trajectories for finding the maximum of the "radio plume"



## Incorporating Known Information

We can incorporate existing information about the environmental process, from sources such as (1) **weather stations**, (2) **satellite imagery**, or (3) **computer models**, into our algorithm.



Incorporating three different types of additional information: temperature from weather stations, satellite image of aerosol optical depth, and a computer model of organic carbon percentage of total aerosol mass. First column shows the truth. Second column is the additional information available to the algorithm before the vehicle collects any data *in situ*. Third column shows a possible trajectory using the *in situ* information only, overlaid on the estimate of the field. Fourth column shows a possible trajectory using both sources of information, overlaid on the estimate of the field.

## Future Work

- three dimensions
- time-varying processes
- multiple vehicles