

# Inferring Release Characteristics From an Atmospheric Dispersion Model

Bruno Sansó

[www.ams.ucsc.edu/~bruno](http://www.ams.ucsc.edu/~bruno)

Department of Applied Mathematics and Statistics  
University of California Santa Cruz

# The Cast

The work presented in this talk was done in collaboration with



Devin Francom, LANL



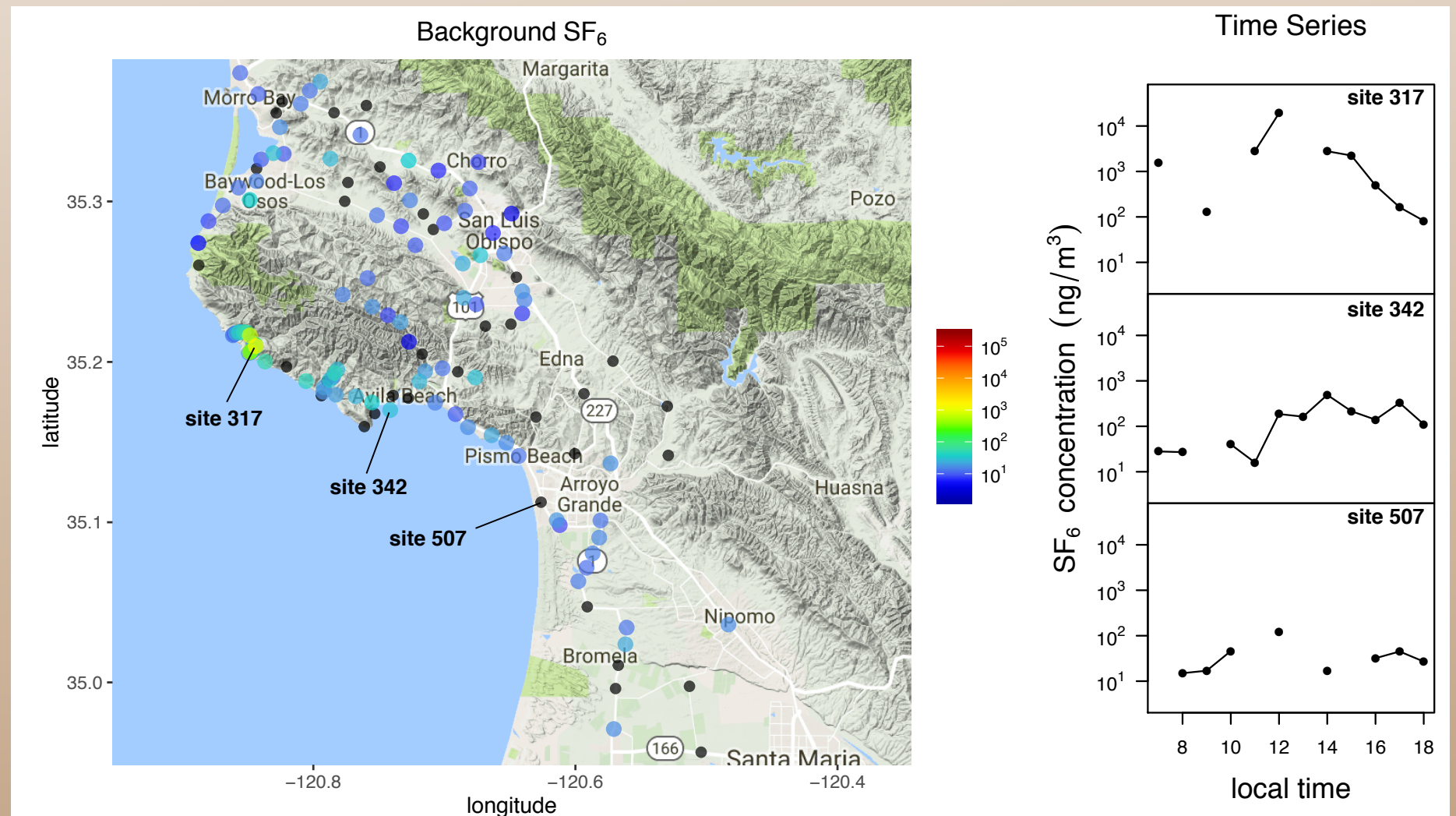
Vera Bulaevskaya, TCC



Donald Lucas, LLNL

Matthew Simpson, LLNL

# Diablo Canyon Nuclear Power Plant

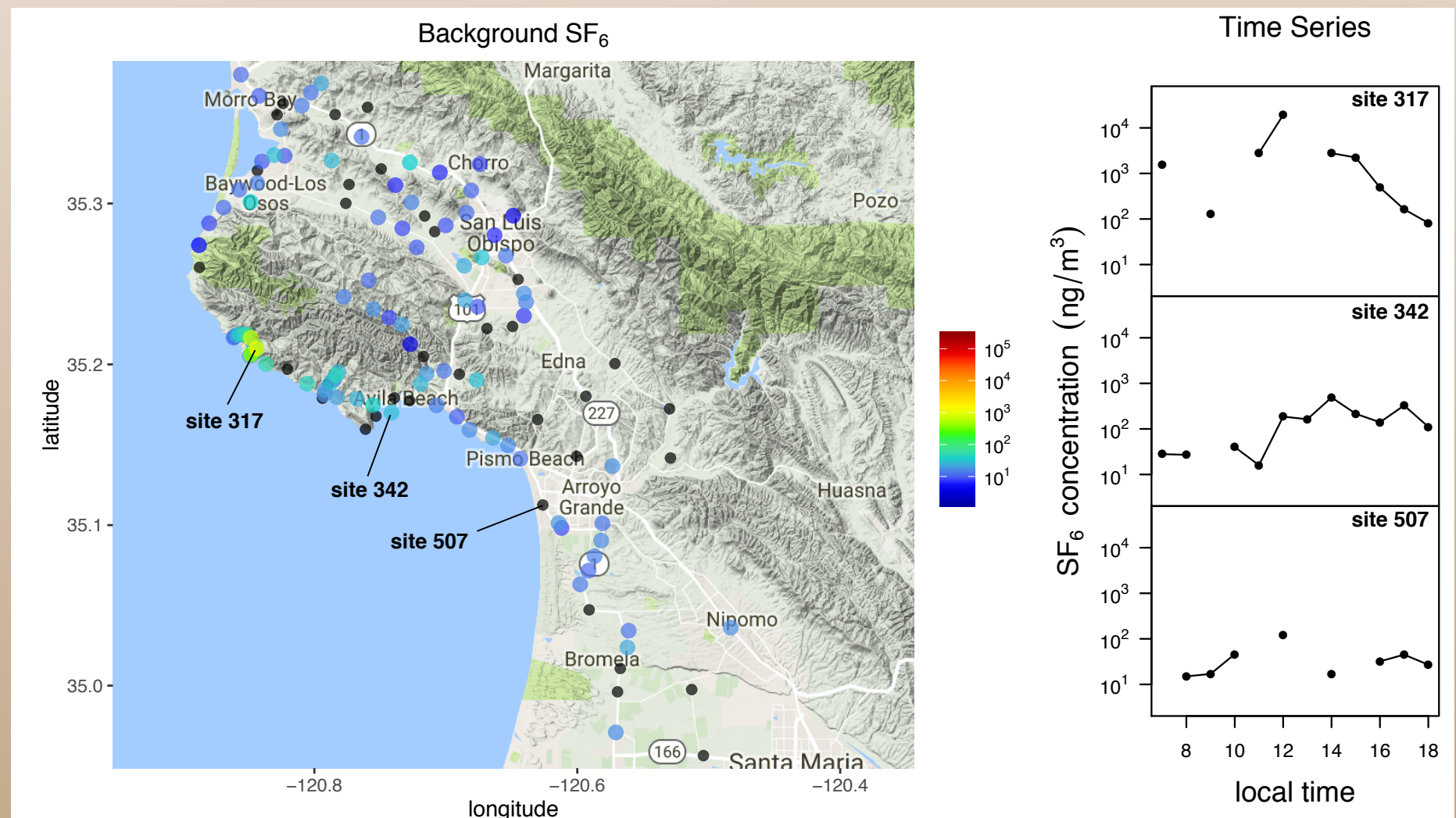




# Diablo Canyon Nuclear Power Plant



146 Kilos of  $\text{SF}_6$  where released from the Diablo Canyon plant on Sept. 4, 1986 for 8 hours, starting at 8:00.



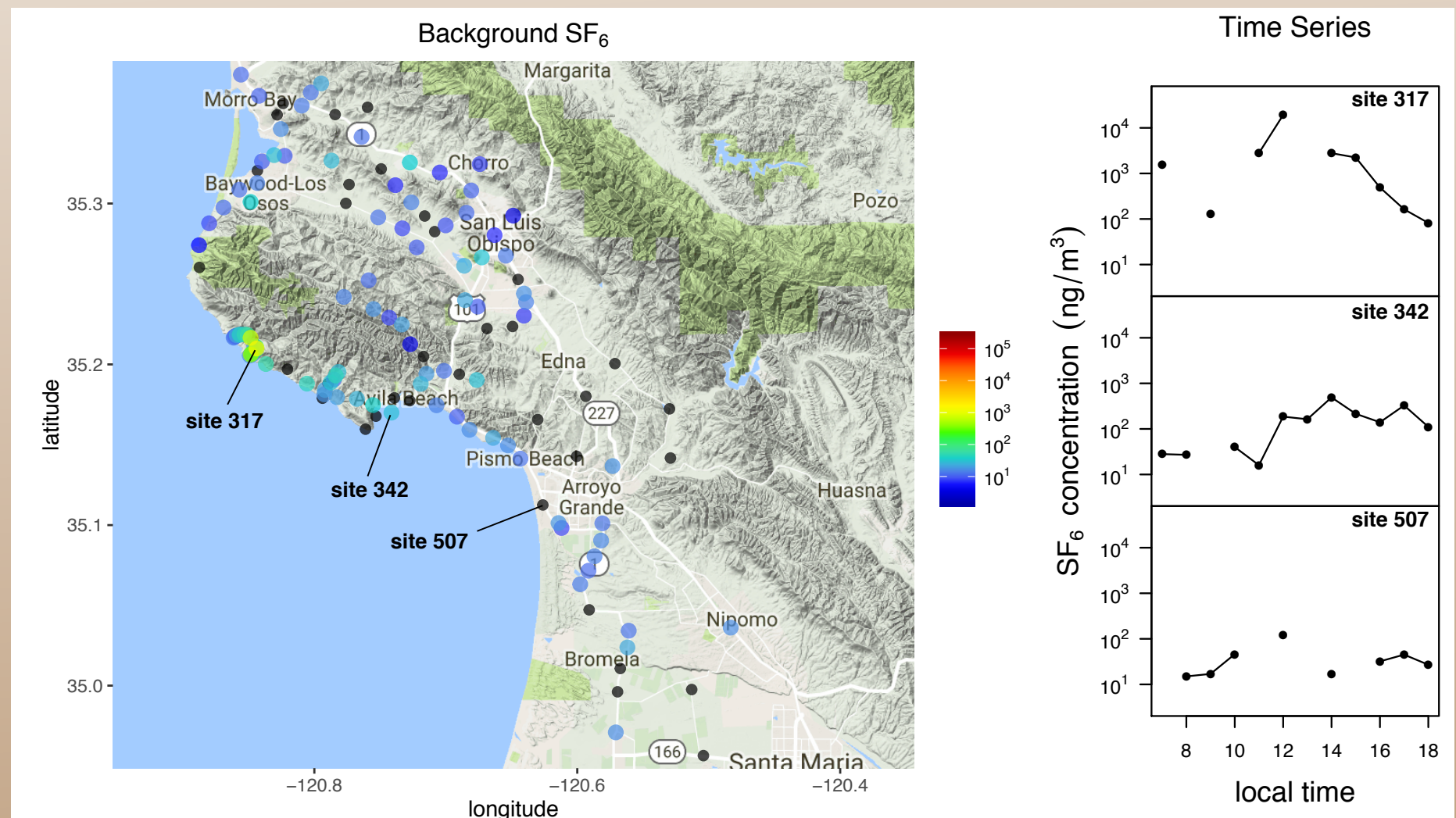


# Diablo Canyon Nuclear Power Plant



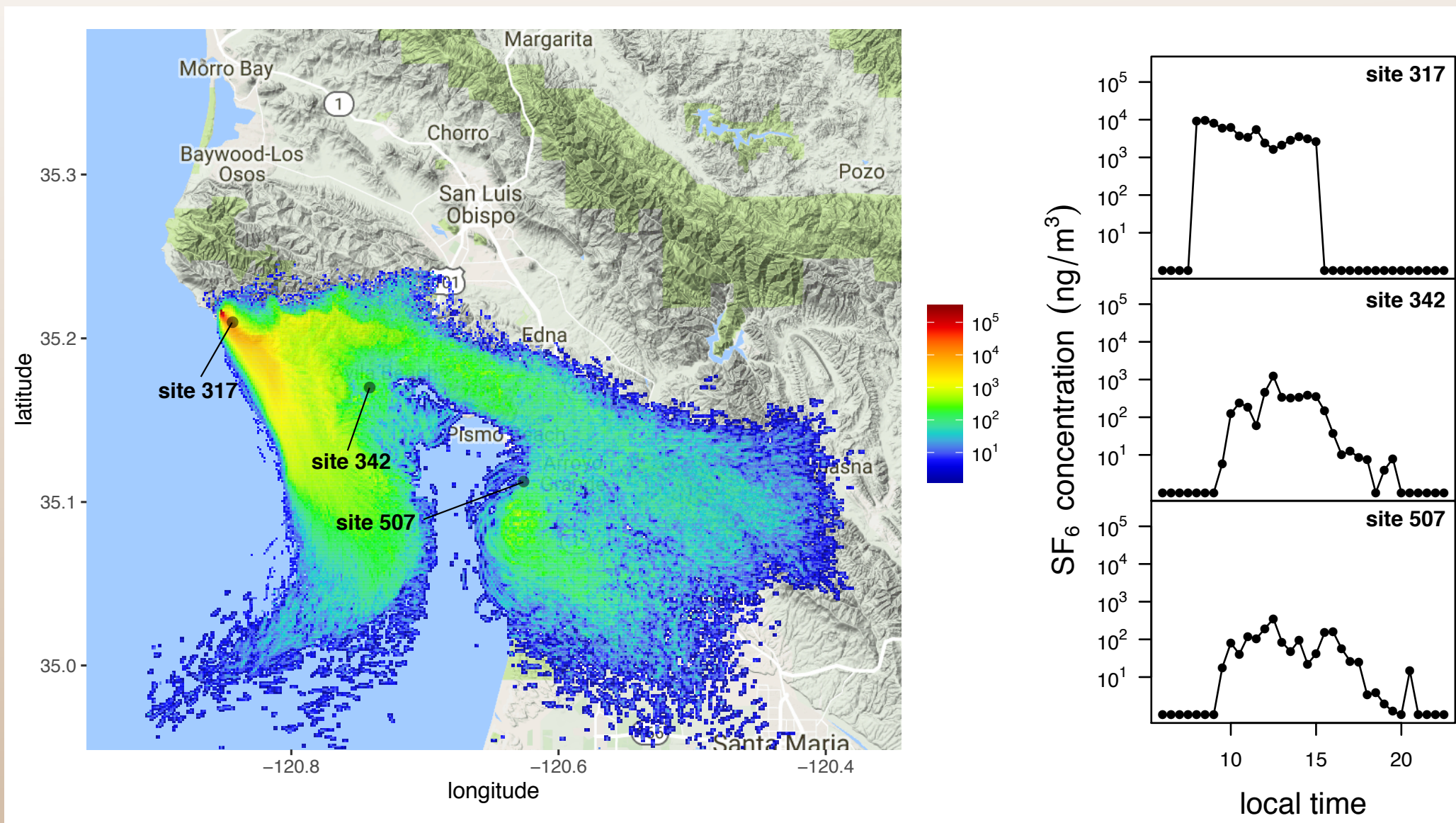
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Air samples were obtained from 7:00 to 18:00 at 150 sites. 24% are missing.





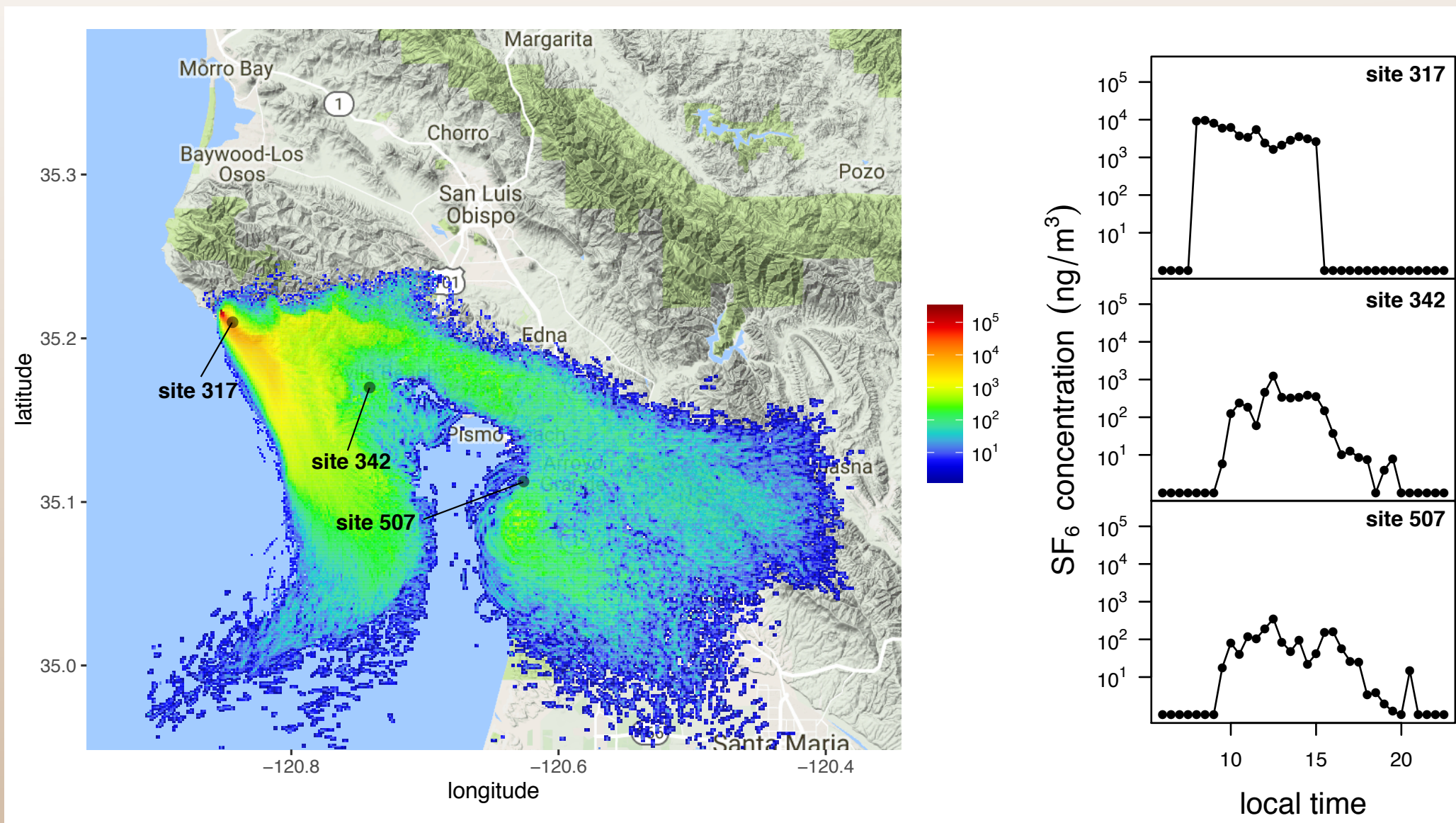
# FLEXPART Simulations



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18,000 different combinations of the 11 input parameters of FLEXPART are sampled from a latin hypercube. These result in 18,000 plumes varying in space and time.

# Input Parameters

## Continuous Input Parameters

Input	Lower Bound	Upper Bound	True Value
Latitude	35.1977	35.2250	35.2111
Longitude	-120.87	-120.83	-120.8543
Altitude	1	10	2
Start Time	7:00	9:00	8:00
Duration	6	10	8
Amount	10	1000	146.016

## Categorical Inputs

Input	Number of values
Pre-release Initialization time	2
Boundary Layer Model	3
Nudging	3
Reanalysis	3
Land Model	3

There are five nested domains for WRF models. Each combination of the five categorical variables produces a different wind field at 300 meters resolution.



# Emulator

We build an emulator for the computer output corresponding to location  $s$ , time  $t$  and input values  $x$ , by using the representation on empirical orthogonal functions

$$y^c(s, t, x) = \sum_{i=1}^k K_i(s, t) w_i(x) + u(s, t)$$

The EOFs are calculated from the model runs

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For the truncation we take a non-Gaussian error

$$u(s, t) \sim Unif\left[y^c(s, t, x_j) - \sum_{i=1}^k K_i(s, t) w_i(x_j), \right. \\ \left. j = 1, \dots, n_x\right]$$

This is important in order to propagate truncation uncertainty



# Estimating the EOF coefficients

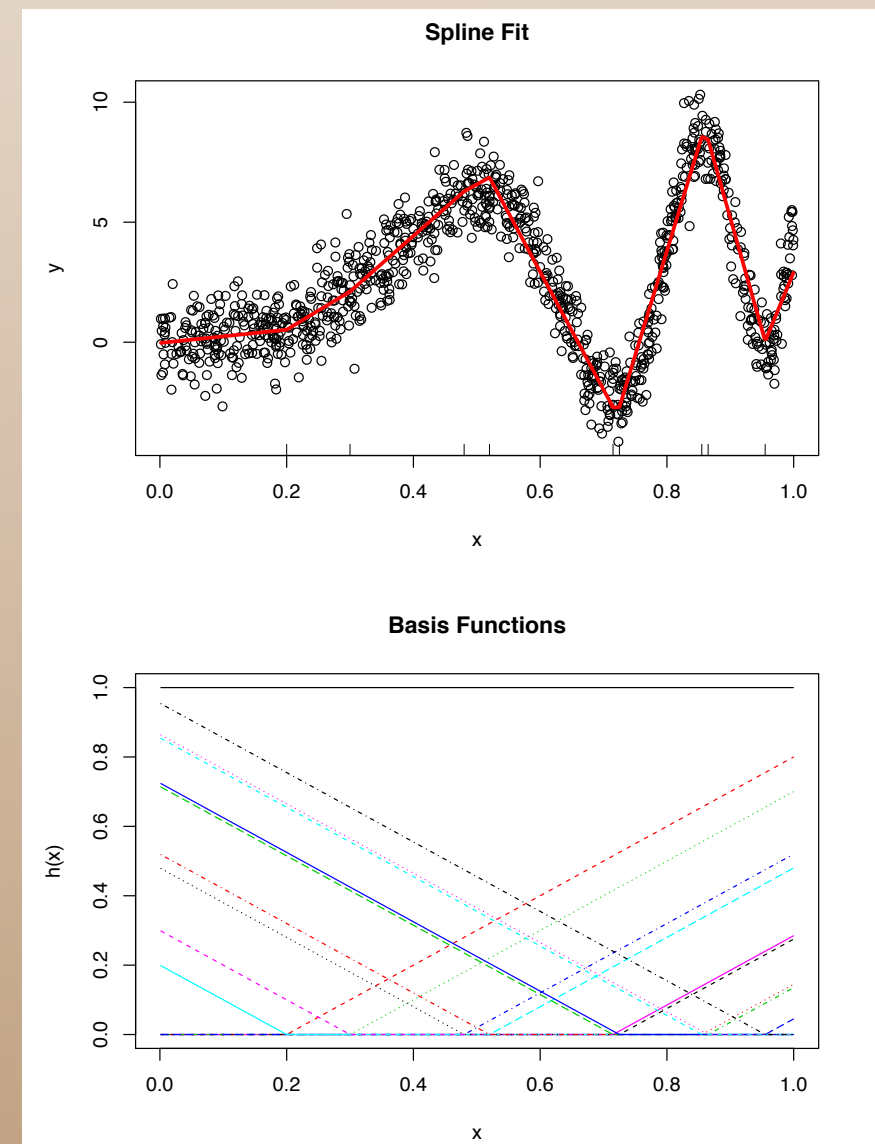
To estimate the coefficients in the EOF we set:

$$w_i(x) = \eta_i(x) + \epsilon_i$$

where

$$\eta(x) = a_0 + \sum_{m=1}^M a_m B_m(x)$$

is a representation on **adaptive spline** basis composed of  $M$  (unknown) terms, that uses products of hockey sticks with varying signs, number of interactions, and unknown knots.



# Categorical and Continuous Inputs

Our application requires the emulator to handle continuous and categorical inputs. Assume that  $x_1$  and  $x_2$  are continuous, and  $x_3$  and  $x_4$  are categorical, then

$$B(x) = [s_1(x_1 - t_1)]_+^\alpha [s_2(x_2 - t_2)]_+^\alpha \mathbf{1}_{x_3 \in C_3} \mathbf{1}_{x_4 \in C_4}$$

where  $t_1$  and  $t_2$  are the knots and

$$s_i = \pm 1$$

and  $C_i$  corresponds to one or more categories of the  $i$ -th variable.



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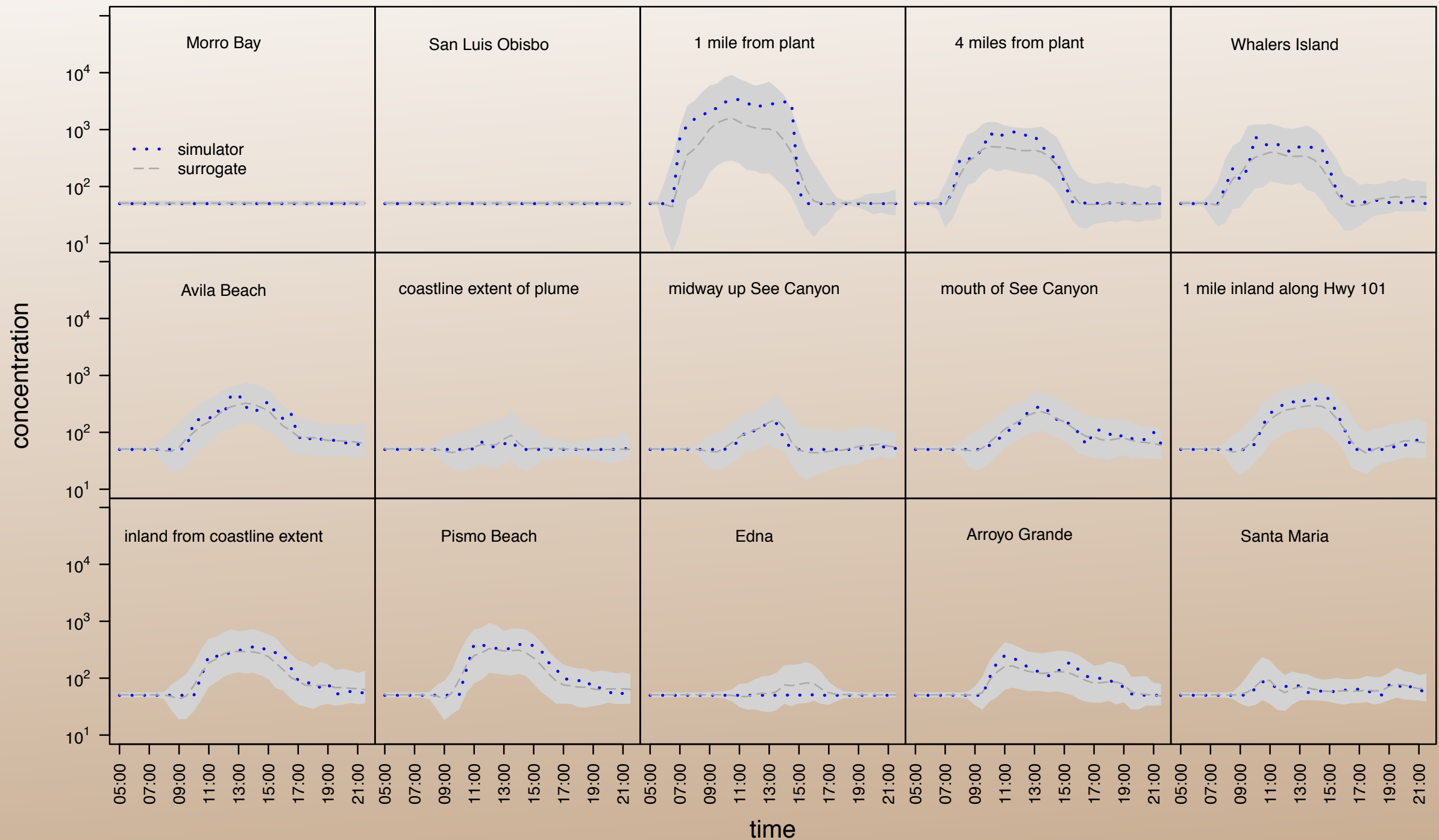
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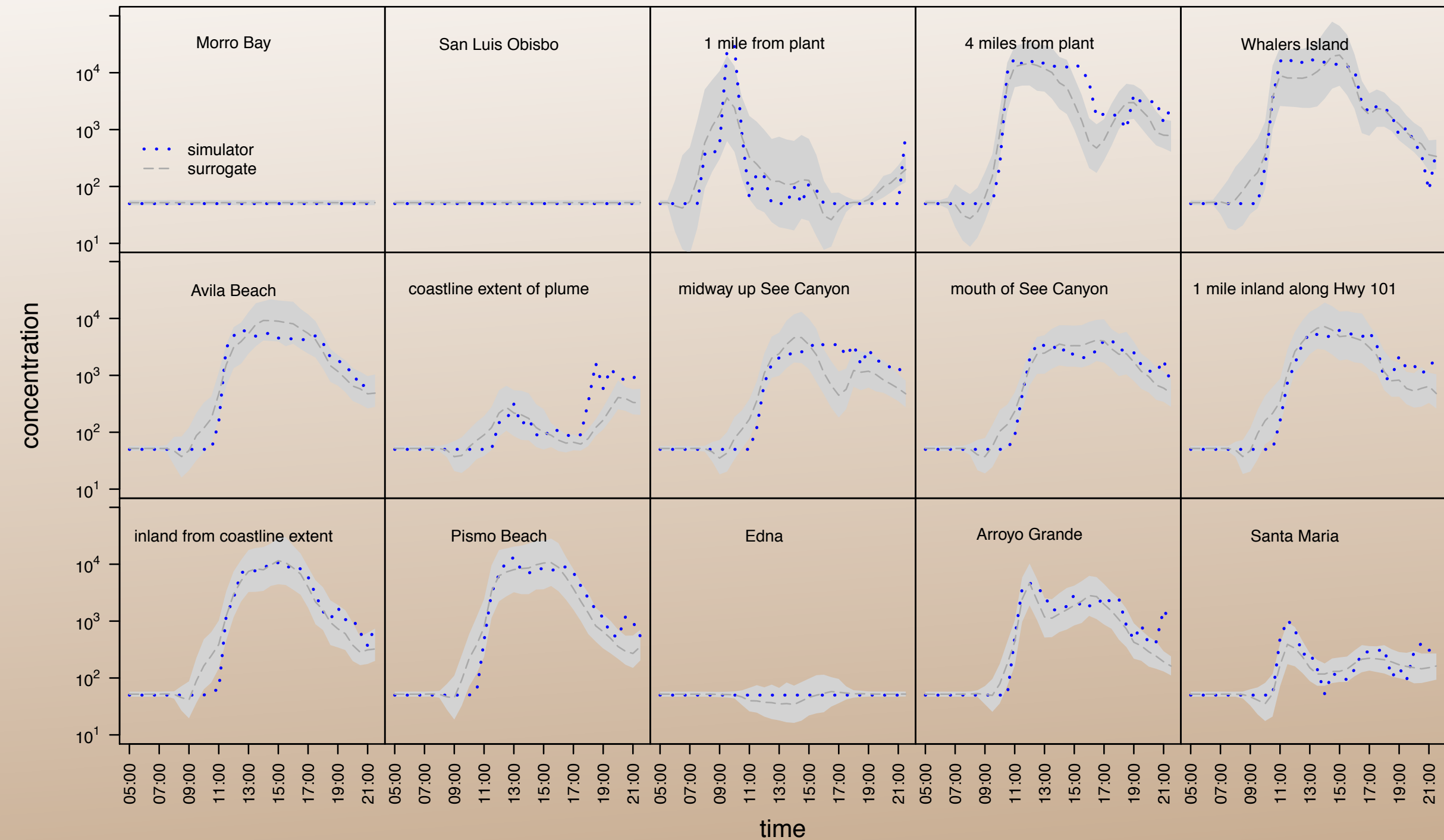
- Allows for basis functions specific to some categories
- Allows for basis functions common to all categories
- Learn from the data the categorical variables in each basis function, if any

# Emulation Performance



Predictions at 15 of the of the 137 locations considered for a held out configuration of the input parameters.

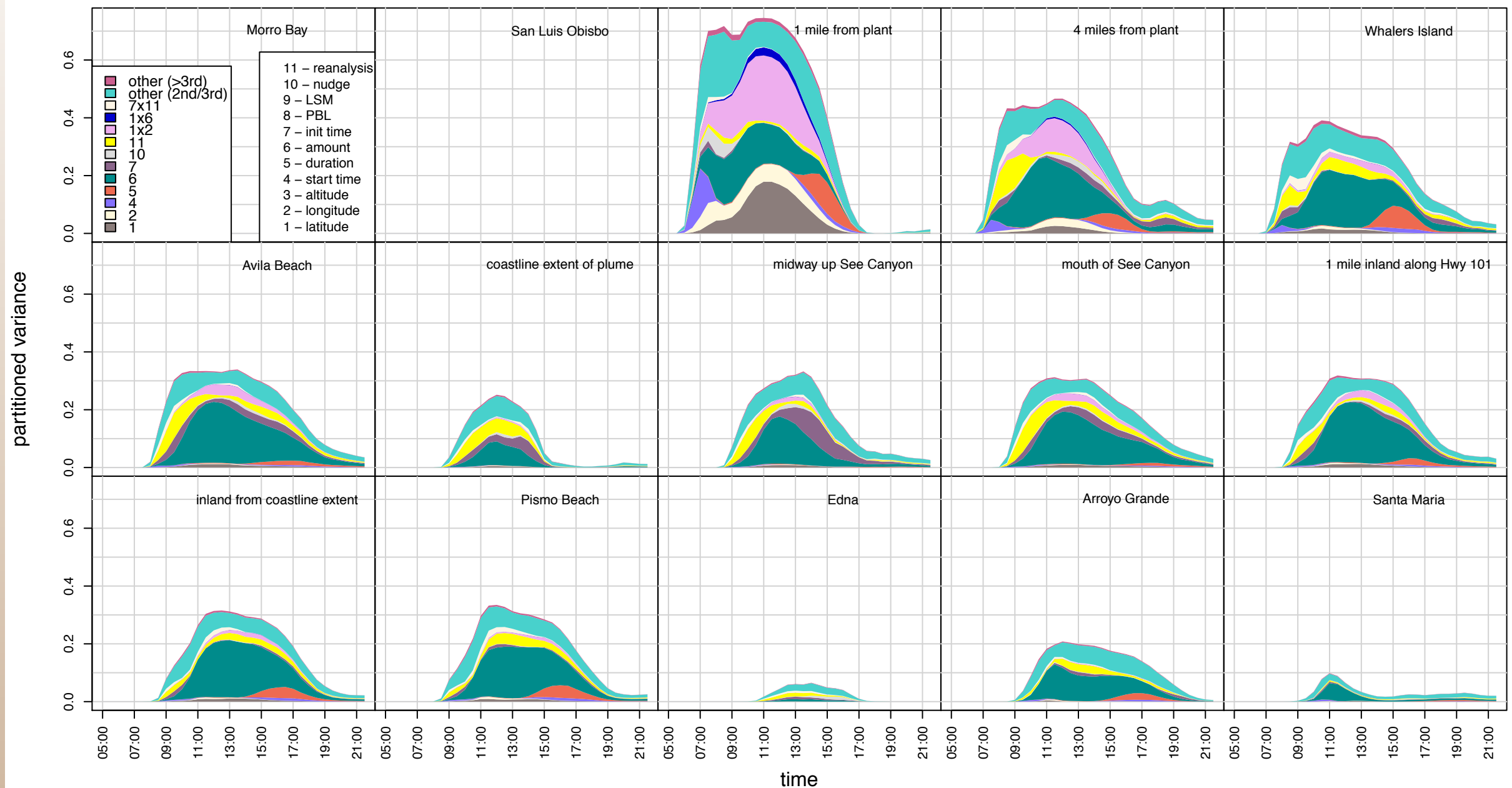
# Emulation Performance



Predictions at 15 of the of the 137 locations considered for another held out configuration of the input parameters.



# Global Sensitivity



Analytic expressions are available for the time and space-varying Sobol coefficients for the different inputs and interactions.

# Calibration: $p(\theta|Y^F, Y^C)$

Observation Equation  
with Gaussian error

$$\overbrace{y^F(s, t)}^{\text{Observations}} = \underbrace{\zeta(s, t)}_{\text{True System}} + \overbrace{v(s, t)}^{\text{Obersv. Error}}$$

System Equation with  
additive discrepancy with  
U[0,2] multiplication  
factor

$$\zeta(s, t) = \underbrace{y^P(s, t, \theta)}_{\text{Best Estimate}} + \overbrace{\gamma \delta(s, t)}^{\text{Discrepancy}}$$

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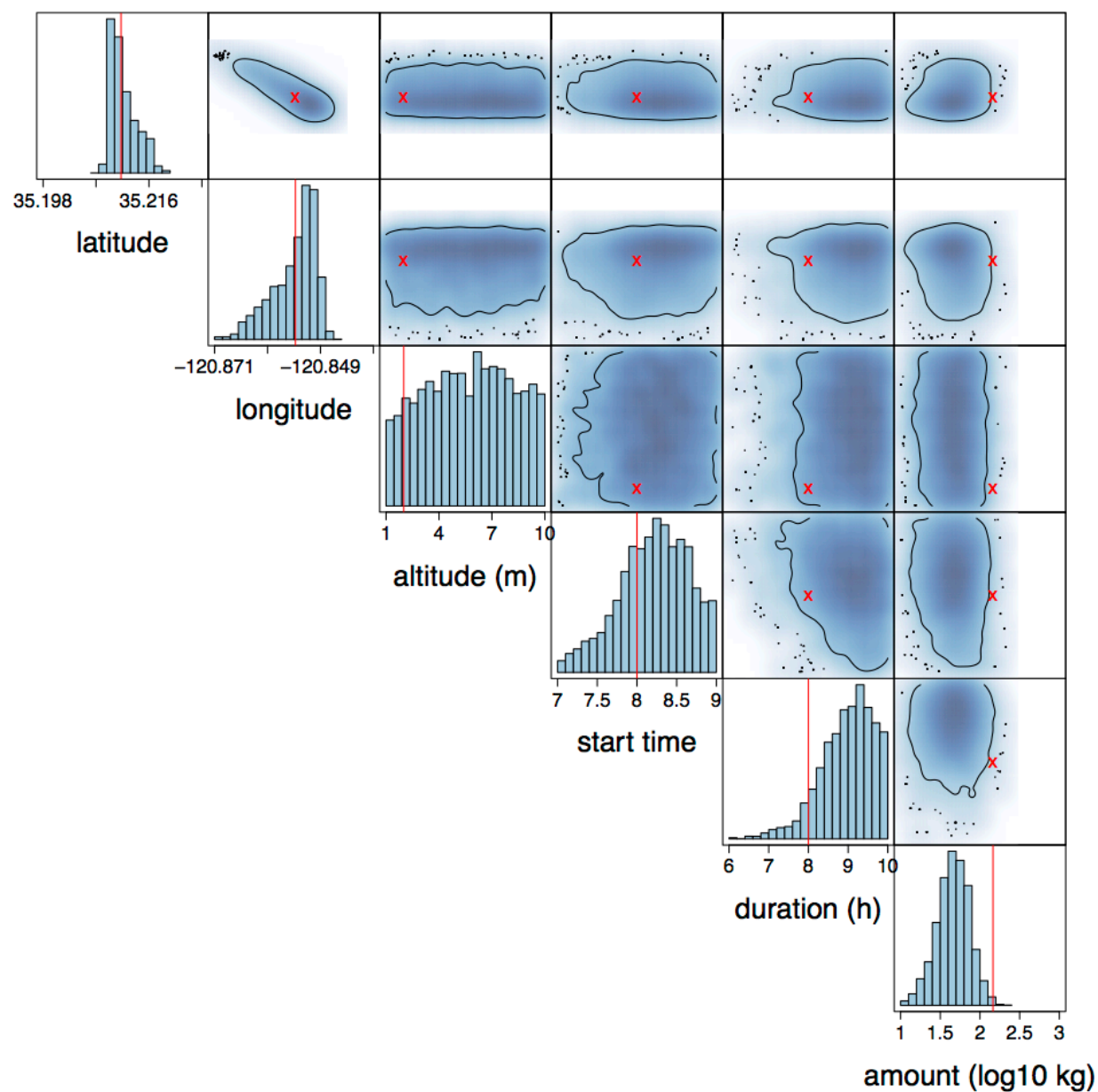
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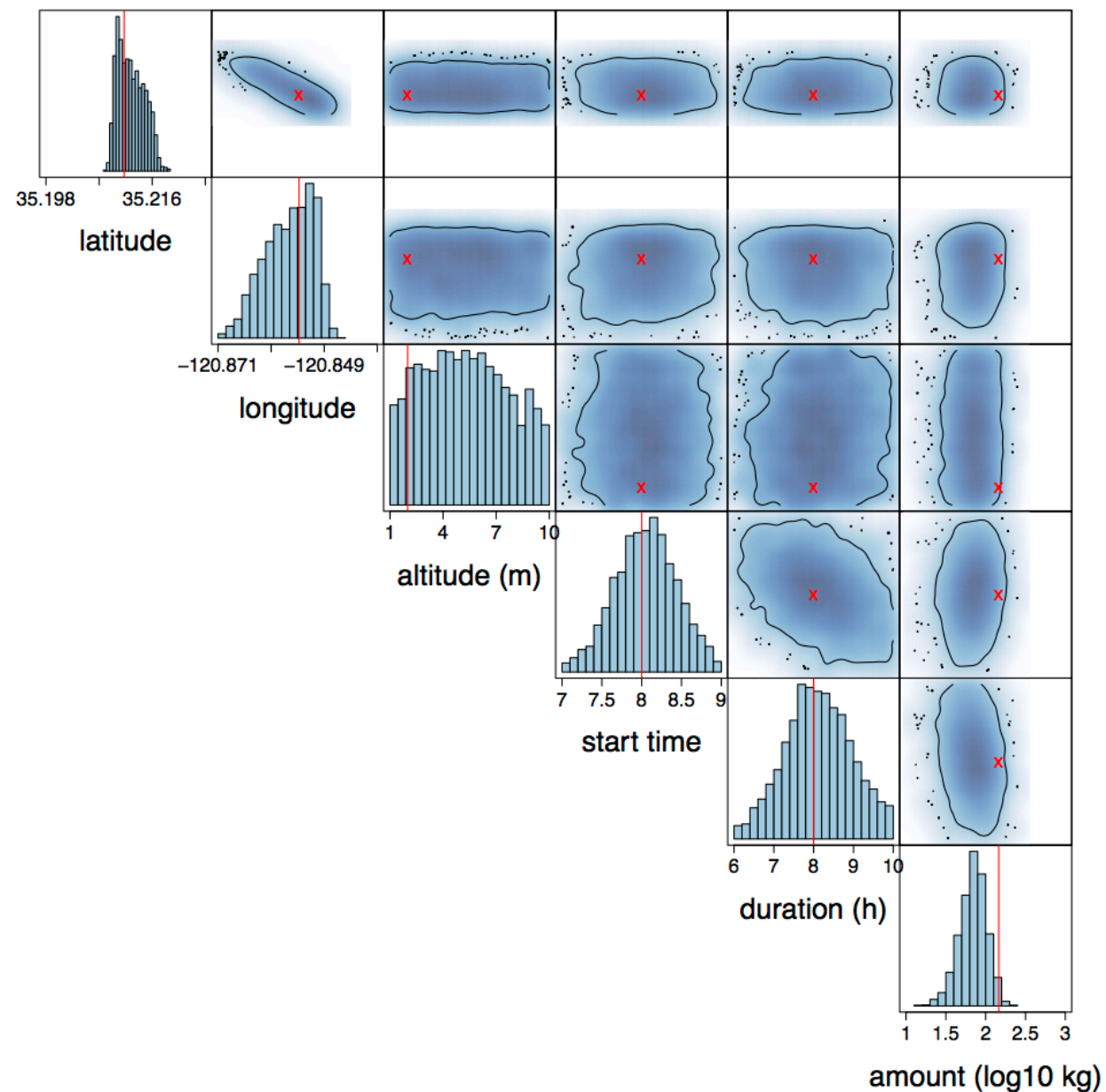
We estimate the discrepancy by fitting the emulator at the prior mean of the inputs. We then fit the discrepancy using adaptive splines (BASS).  $\gamma$  provides information about the relevance of the discrepancy.



# Posteriors for Continuous Inputs

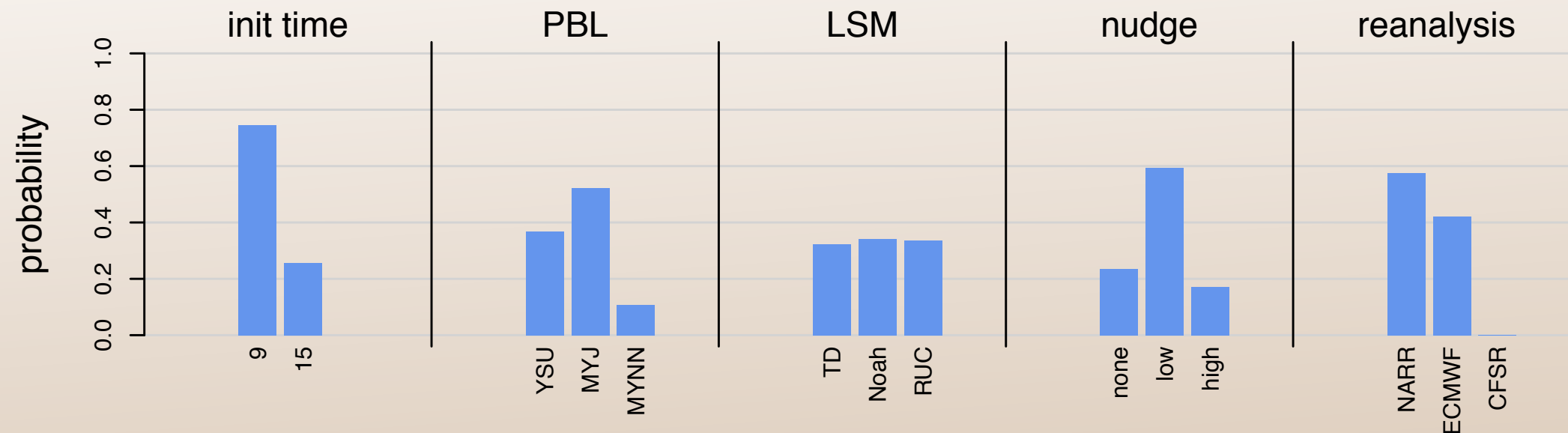


No discrepancy

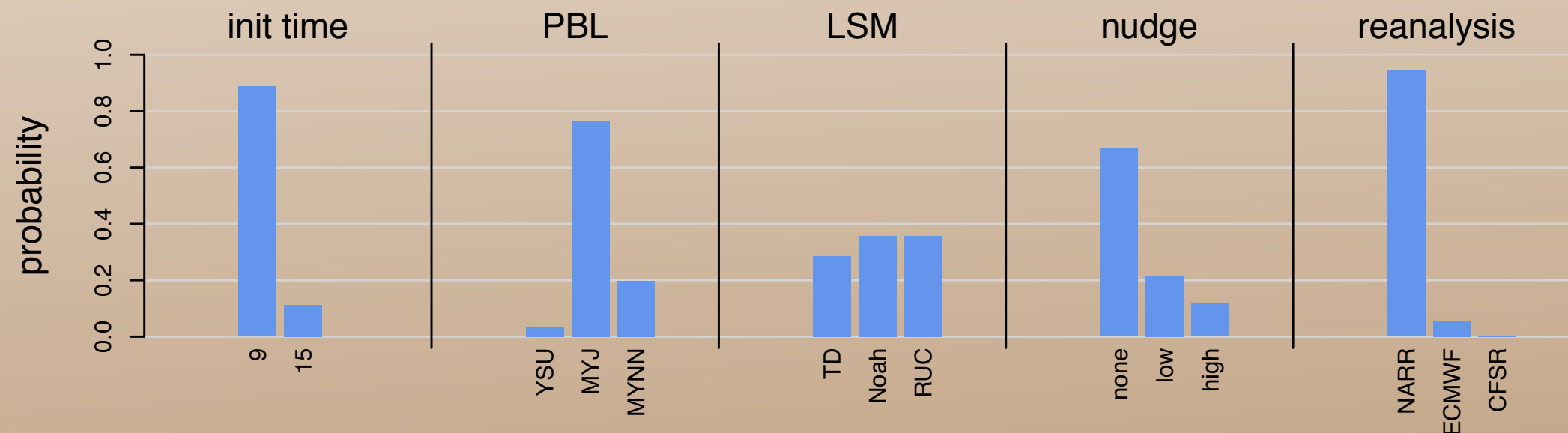


Discrepancy

# Posteriors for Categorical Inputs

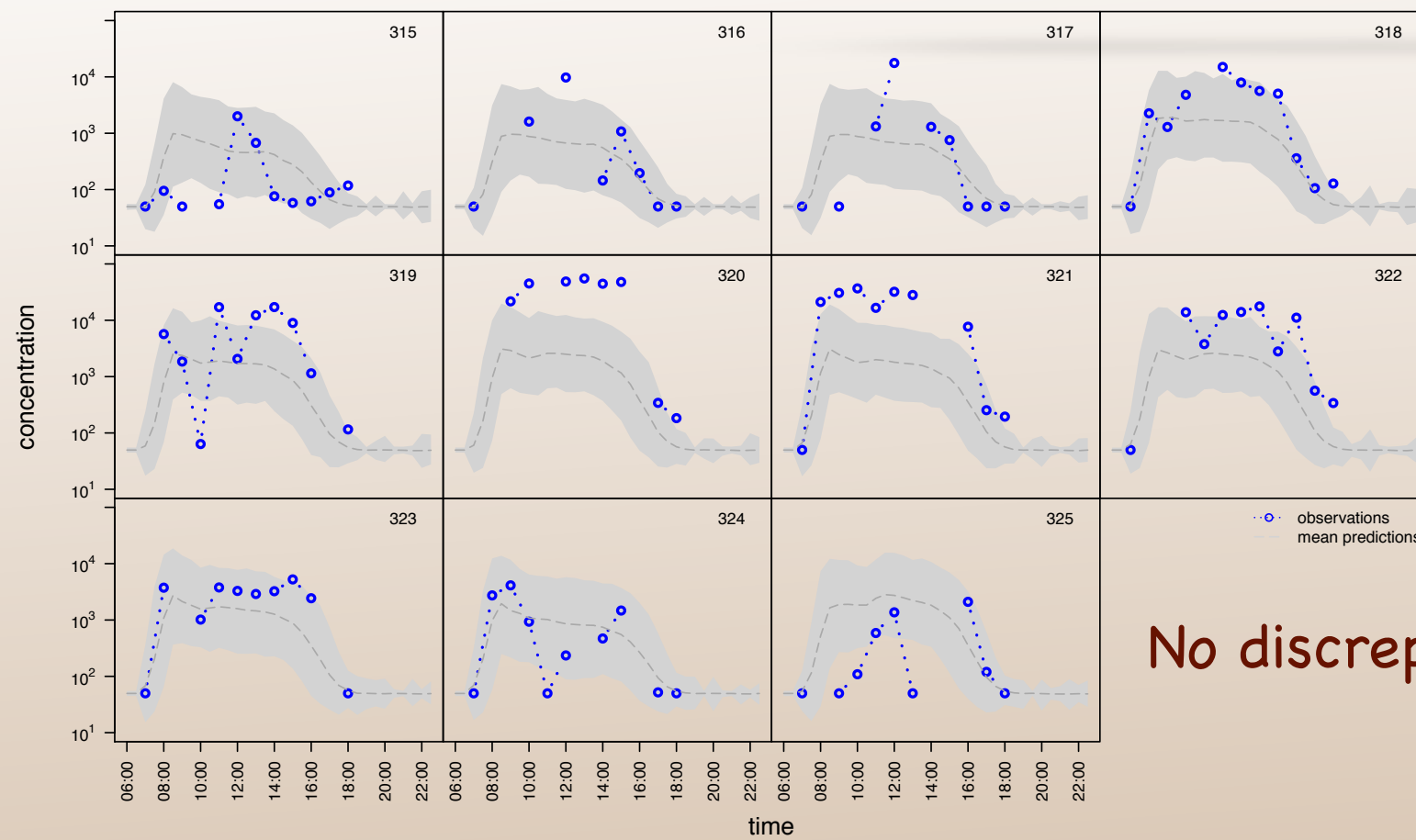


No discrepancy

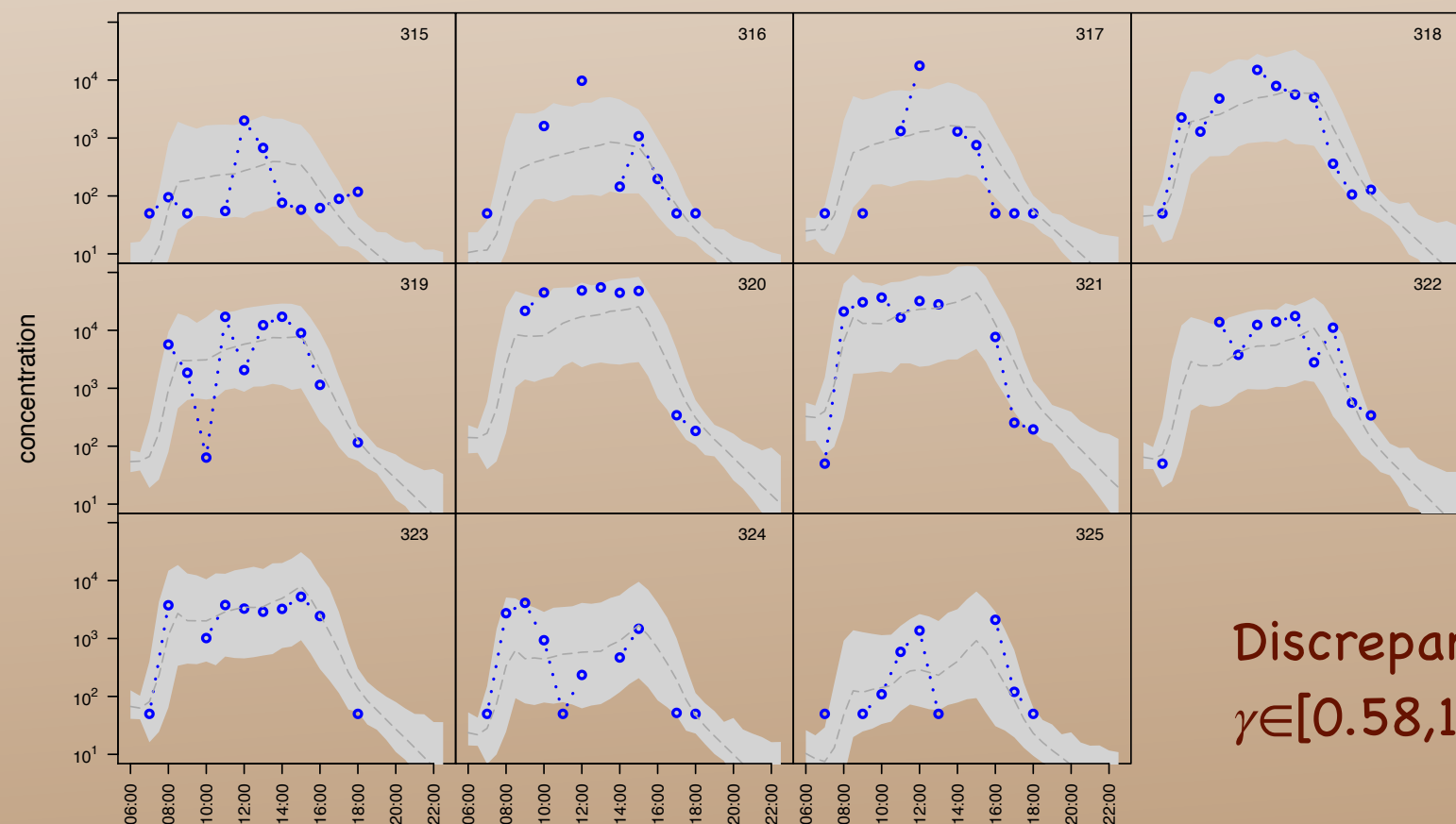
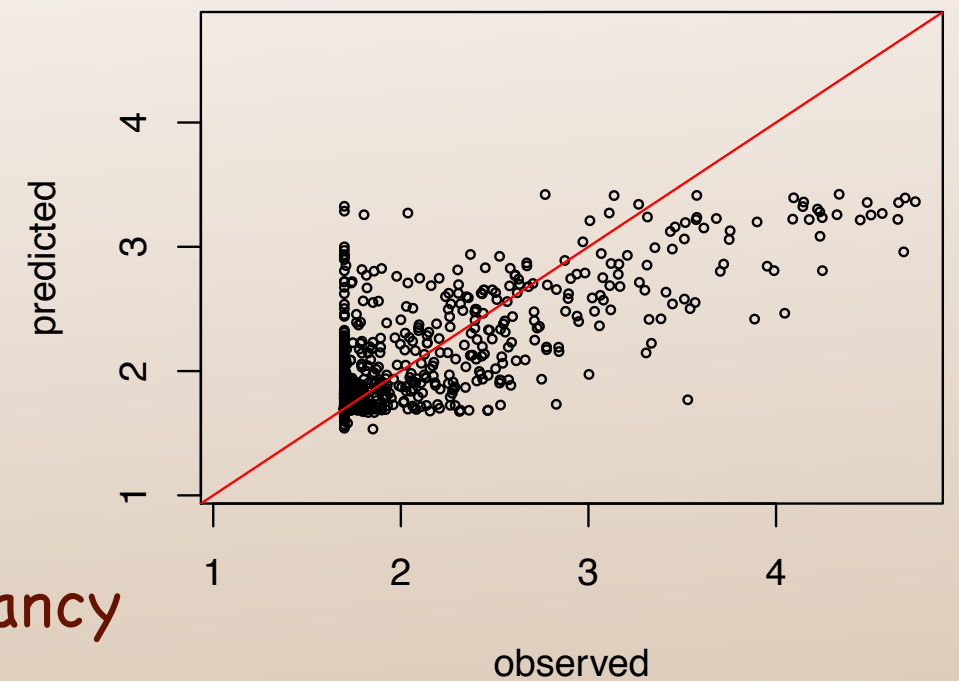


Discrepancy

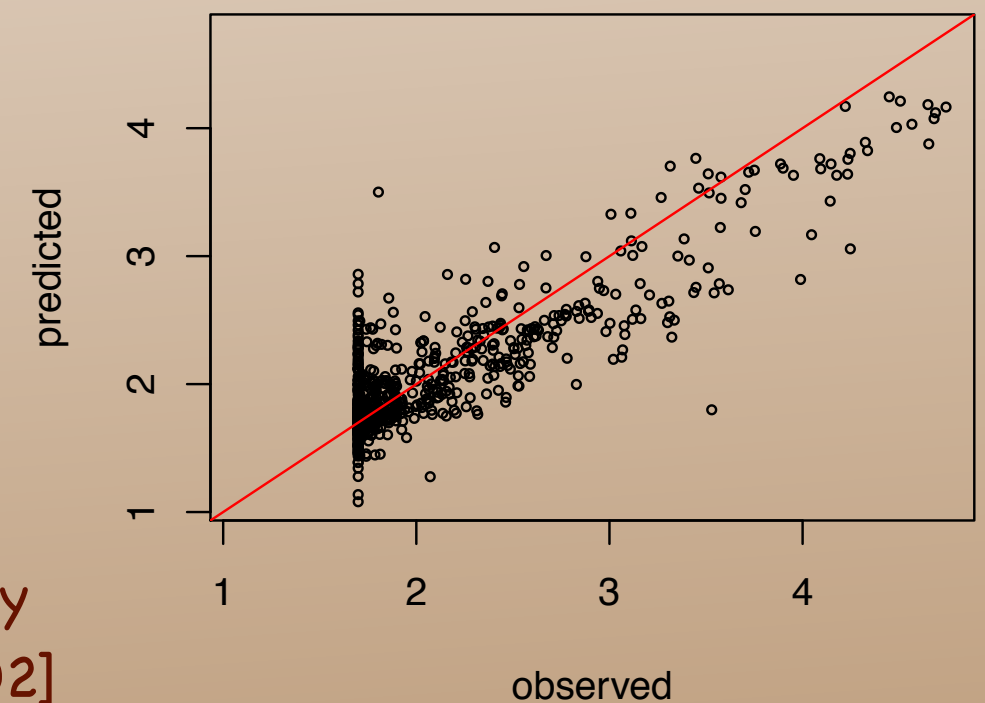
# Calibrated Predictions



No discrepancy

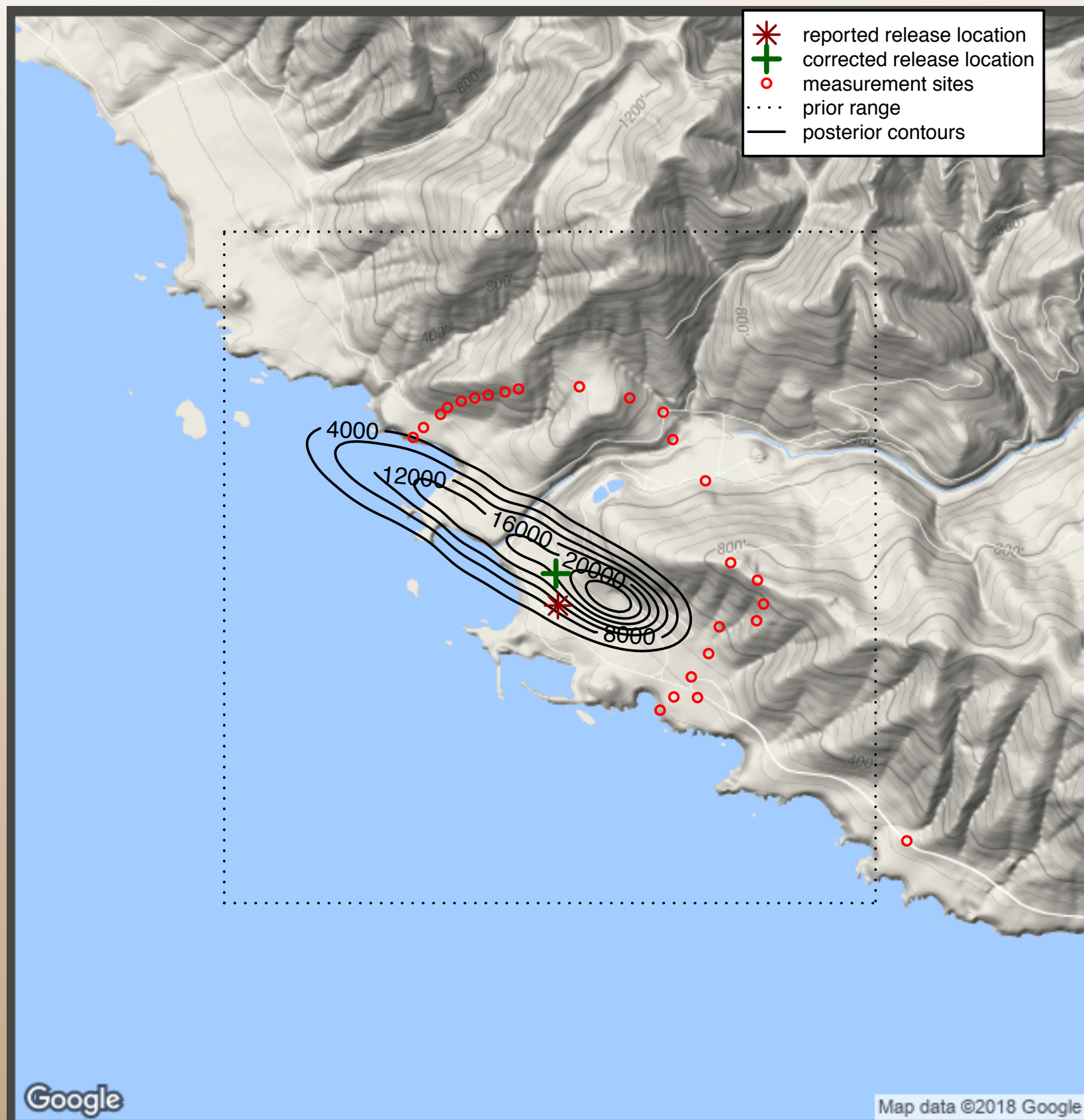


Discrepancy  
 $\gamma \in [0.58, 1.02]$



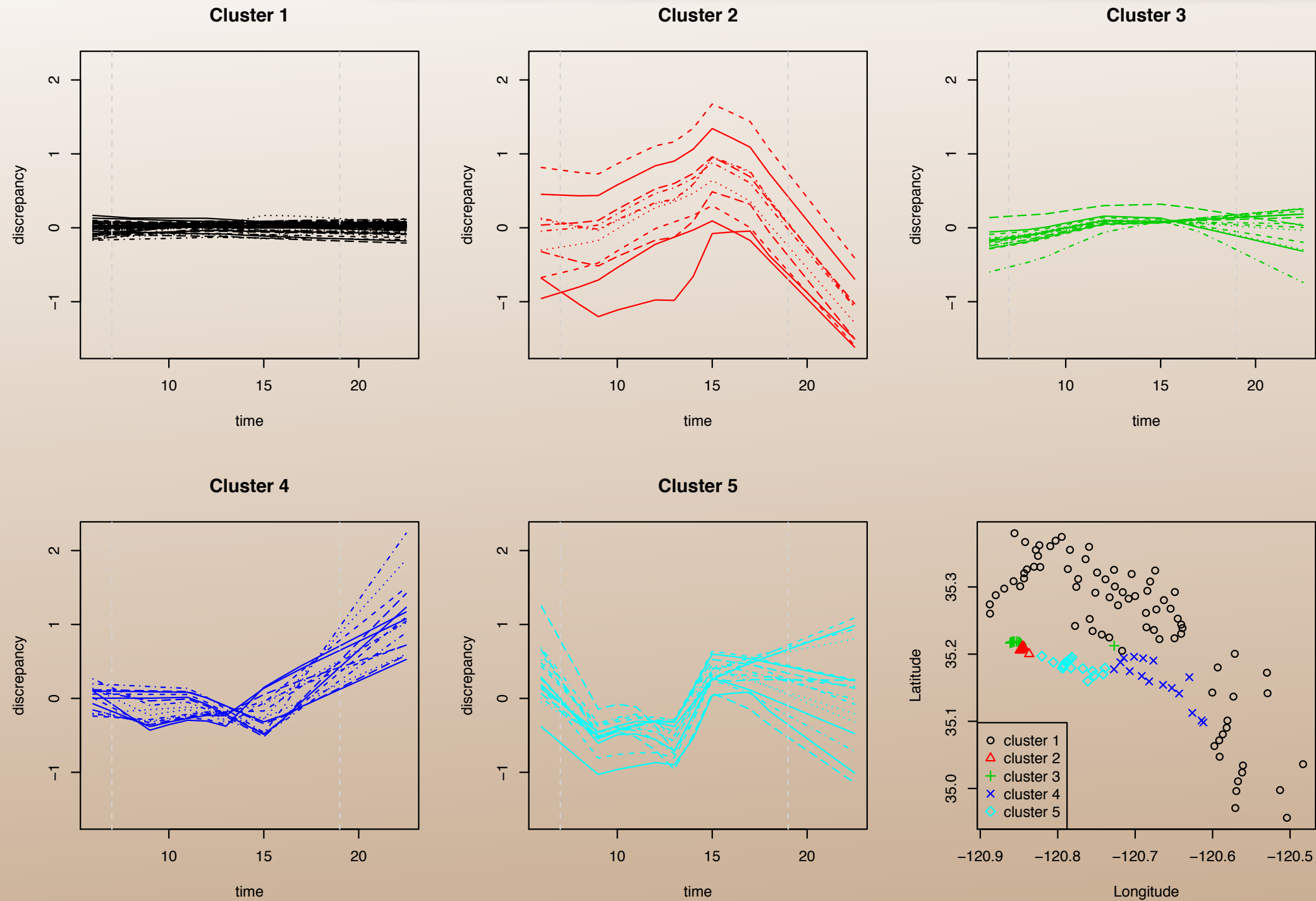


# Calibrated Release Location



The release location was originally mis-reported. Our posterior distribution reveals that a second source of information corresponds to a much more probable location

# Analysis of Discrepancies



Clusters of discrepancy curves identifying clear location patterns.

# Conclusions



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- Our method scales well to large amounts of data, providing accurate emulation and prediction.
- Our method can handle continuous and categorical inputs.
- We able to perform a time and space-varying sensitivity analysis of the inputs based on accurate analytic expressions for the global sensitivity coefficients.
- The method uses a fully probabilistic approach that allows to account for all sources of variability and provides a coherent quantification of the uncertainty.

# References

# References

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