Statistical analysis of convection in variable stars using realistic hydrodynamic simulations

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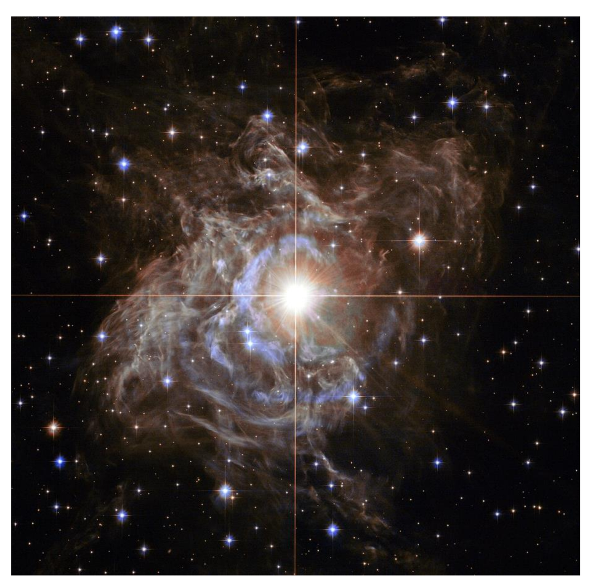
Abstract

Providing a description of mixing in stellar interiors requires statistics that draw on large data sets and long-time data series. This is particularly true for Cepheid variable stars, evolved stars of intermediate mass that experience radial pulsations. The fact that these stars experience periodic variations in their structure makes it more difficult to obtain converged statistics for intermittent processes like turbulent convection. In this contribution, we discuss the challenges in obtaining statistics for convective boundary mixing for interior convective shells in simulations of Cepheids with masses that range between 6 and 9 solar masses produced with the MUSIC code. We produce five realistic two-dimensional hydrodynamic simulations of the interior convective regions of these stars. We examine statistics for convection and convective overshooting, based on approximately 7 TB of data. We apply extreme value theory to the maximal extent of convective overshooting over time to investigate how convective plumes evolve. We develop statistical diagnostics to study how convective patterns relate with the mass and luminosity of these stars. This work serves as a basis for developing new statistical analysis for larger, three-dimensional simulations of the whole star that include radial pulsations, a project that we will pursue with LLNL's most advanced multi-physics Arbitrary Lagrangian-Eulerian code Marbl.

Cepheid variables – A cosmic distance indicator

Improving the theoretical description of Cepheids in hydrodynamic simulations would allow us to improve stellar evolution models for these complex stars



RS Puppis, one of the brightest known Cepheid variable stars in the Milky Way galaxy (Hubble Space Telescope) [2]

A cosmic benchmark

- Period-luminosity relationship: **the Leavitt Law** [3]
- Scale extra-galactic distances and establish the Hubble constant
- Study specific properties of the Galaxy (chemical compositions, spiral structure of the Milky Way)

Our theoretical knowledge remains incomplete

- Mass-luminosity relation poorly constrained
- **Mass discrepancy problem**: typical disagreement of 10 – 20% between stellar evolution models and stellar pulsation calculations

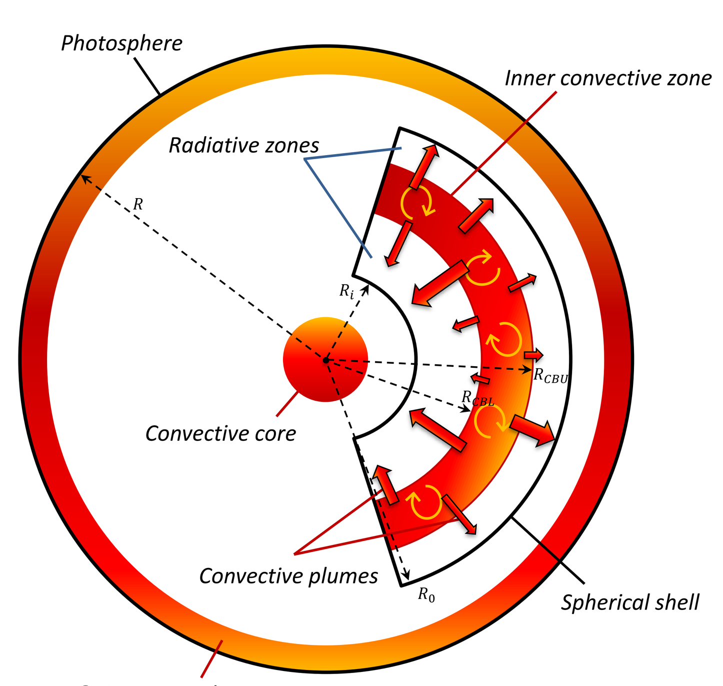
Science Objectives

By producing a large enough data set of realistic hydrodynamic simulations, we can improve our prediction of convective overshooting using statistical tools

Inner structure of Cepheids

- Convectively unstable layers
  - A small **convective core**
  - A **convective shell**
  - A small **sub-surface convective envelope**
    - ➢ *Energy is primarily transported by fluid mixing mechanisms*
- Separated by convectively stable regions (**radiative zones**)
  - ➢ *Energy is primarily transported by radiation and conduction*

Overshooting ➡ Convective plumes cross convective boundaries and penetrates the radiative zones



Inner structure of a Cepheid and the simulated spherical shell

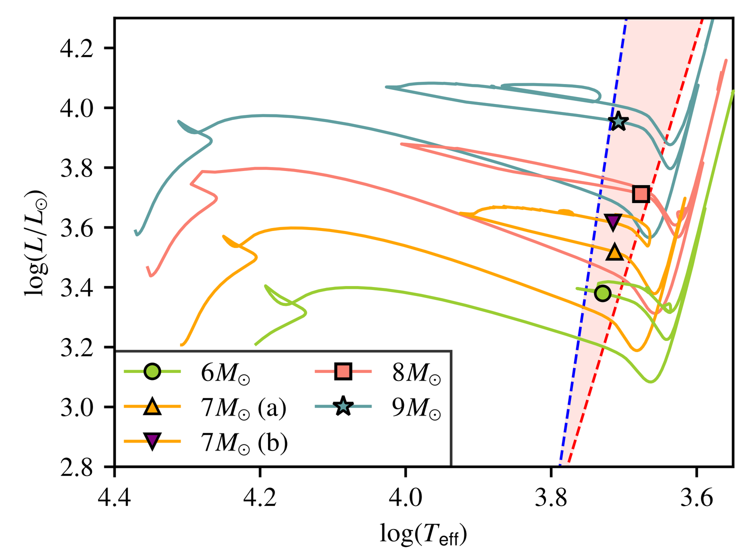
Methodology and numerical simulations set-up

Notation
$M_\odot$: mass of the Sun

1. Hydrodynamical simulations with MUSIC

Five simulations of five Cepheids

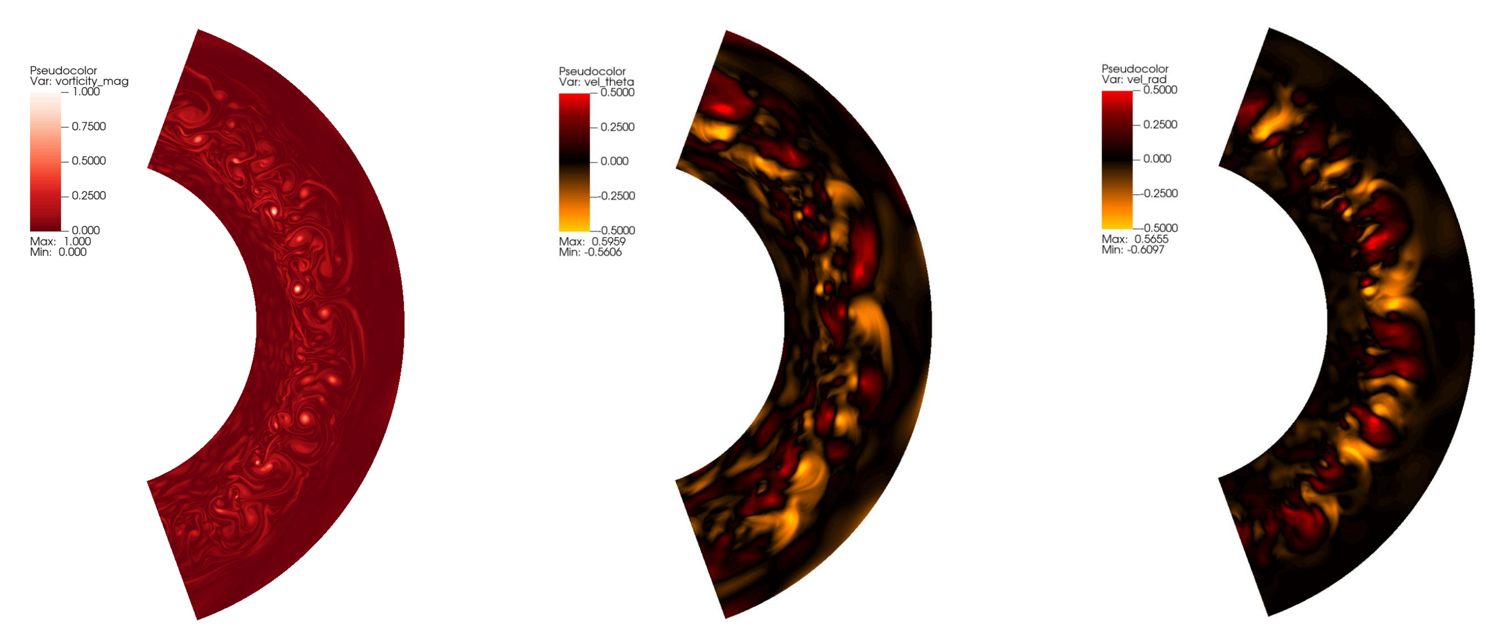
- 4 different masses: 6, 7, 8 and 9 $M_\odot$
- Resolution: $r \times \theta = 1024 \times 1024$
- The 7 $M_\odot$(b) and 8 $M_\odot$ simulations are not fully analyzed yet
- Each simulation covers approximately 10 pulsation periods
- Each simulation produces one file of raw data per time step
- Post-processing files need to be produced to perform statistical analysis of the data set



Hertzsprung-Russell diagram for the 6, 7, 8 and 9 $M_\odot$ stars. The symbols are the simulated Cepheids

Simulation name	Number of raw files	Size of raw data	Number of post-processing files	Size of post-processing data
ceph6	5,496	171.8 GB	21,984	3.2 GB
ceph7a	79,921	2.4 TB	321,020	46.9 GB
ceph7b	113,264	3.5 TB	453,056	66.4 GB
ceph8	14,532	454.4 GB	56,246	8.3 GB
ceph9	12,370	386.8 GB	49,480	7.3 GB
Total	**225,583**	**7 TB**	**901,785**	**132.1 GB**

Size of the raw data sets produced by each MUSIC simulation, and number of diagnostics needed for each data set.

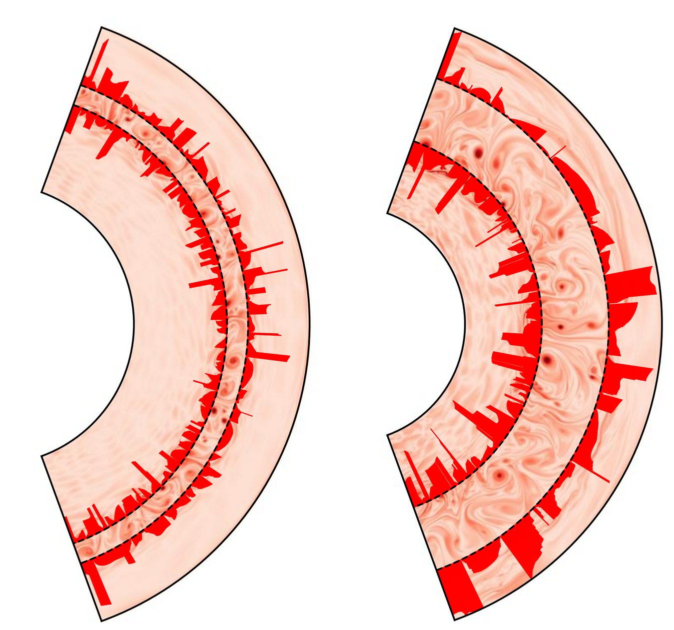


Visualization of the vorticity magnitude (**left**), angular velocity (**middle**) and radial velocity (**right**) for the 7 $M_\odot$ (b) Cepheid.

2. Characterization of convective overshooting

- At each time step and angular position: radial position at which the kinetic energy flux is null
- Stellar evolution models often use a single ***overshooting length*** to account for the effects of convective overshooting

➡ *How can we define a unique overshooting length that will accurately characterize the highly time and position dependent data we produce?*

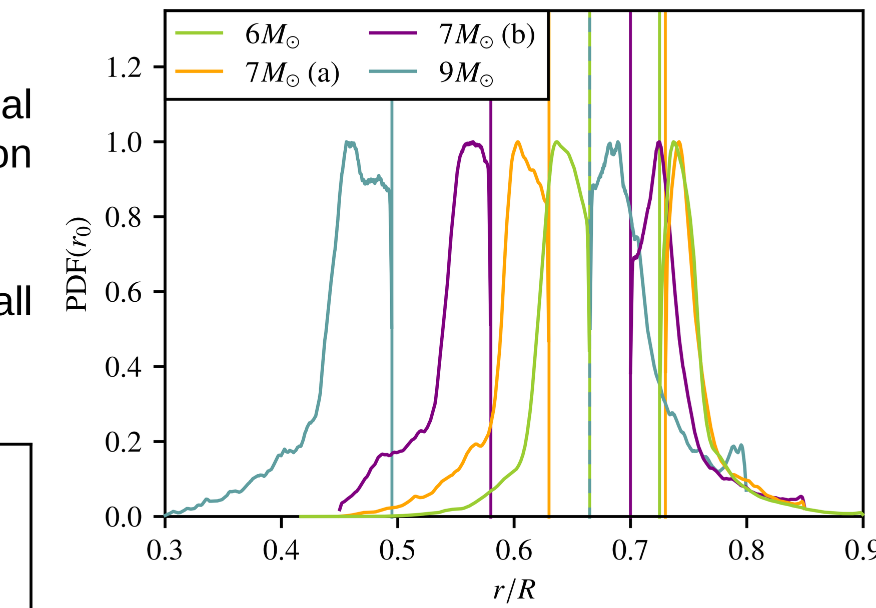


Angular structure of the penetration layer at an arbitrary time for the 6 $M_\odot$ (left) and 9 $M_\odot$ (right) Cepheid

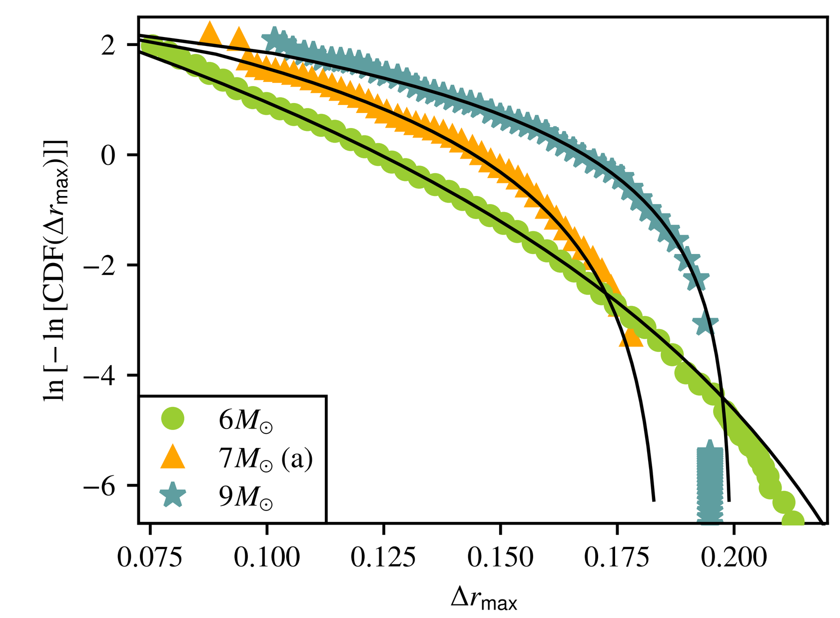
Statistical analysis of large data sets

1. Extreme value theory

- r_0: radial position at which the vertical kinetic flux is null at a given position and time
- Probability density function of all plumes r_0



Probability density functions of the overshooting depth of plumes for simulations ceph6, ceph7a, ceph7b and ceph9



Cumulative density functions of the maximal extent of the penetration length Δr_{max} for simulations ceph6, ceph7a, and ceph9, and best fit with a Weibull distribution (below the convective shell)

- Heavy tail in the PDF of all plumes: influence of the strongest plumes that penetrate further in the radiative zone
- Δr_{max} : maximal extent of the penetration length at a given time

➢ **Weibull distribution:**

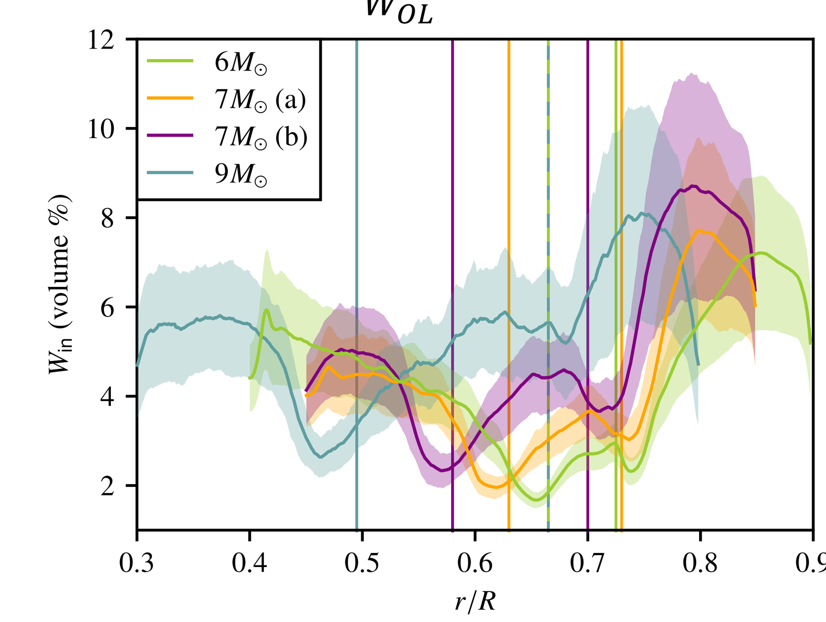
$$F(\Delta r_{max}) = \exp\left[-\left(1+\xi\left(\frac{\Delta r_{max}-\mu}{\sigma}\right)\right)^{1/\xi}\right]$$

▪μ: location parameter = overshooting length

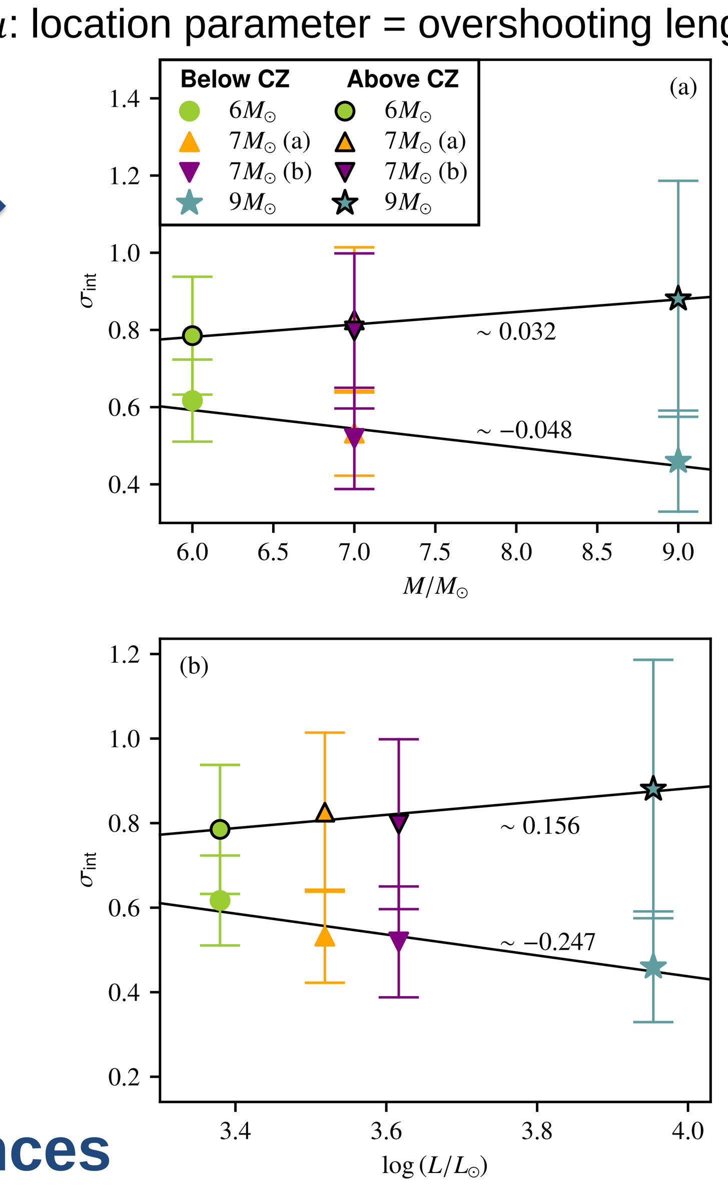
2. Two-point statistics

- We directly look at the average width of plumes moving away from the convective boundaries
- We build a non-dimensional number to compare with mass and luminosity ➡

$$\sigma_{int} = \frac{W_{CZ}}{W_{OL}}$$



Average width of inflowing plumes for the simulations ceph6, ceph7a, ceph7b and ceph9. The shaded area indicates one standard deviation around the mean value.



References

[1] Pratt, J., et al. "Extreme value statistics for two-dimensional convective penetration in a pre-main sequence star." *Astronomy & Astrophysics* 604 (2017): A125.

[2] NASA: https://apod.nasa.gov/apod/ap080212.html

[3] Leavitt, Henrietta S., and Pickering, Edward C. "Periods of 25 Variable Stars in the Small Magellanic Cloud." *Harvard College Observatory Circular*, vol. 173, pp. 1-3 173 (1912): 1-3.

Statistical analysis of Cepheid convection is a key step to improving stellar evolution models and eventually contributing to solving the mass discrepancy problem.

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